Homework 15 (Due Tuesday, April 10th)

1. For the Ising model, mean field theory tells us that the average value of s is found by solving the transcendental equation,

$$\langle s \rangle = \tanh(\langle s \rangle T_c / T)$$

where T_c is the critical temperature. Solve this equation numerically and graph $\langle s \rangle$ versus T/T_c for at least twenty values of temperature from $T/T_c = 0.01$ to 2. You can use any standard root finding method (e.g., bisection, Newton's method) or a built-in routine (e.g., MATLAB's `fzero`) to solve the transcendental equation. Your graph should look something like Fig. 12.7 or Fig. 12.10 (for $H = 0$). **Submit a copy of your program or other documentation describing how you generated your graph.**

1. Consider a lattice made up of alternating A and B sites, such that each spin on an A site interacts only with spins on neighboring B sites, and in such a way that the interaction energy is least when the spins are antiparallel. This is a model of anti-ferromagnetism. The energy is given by

$$E(s_{a,1}, \ldots, s_{a,N/2}, s_{b,1}, \ldots, s_{b,N/2}) = \frac{1}{2} \sum_i \sum_j \epsilon_{ij} s_{a,i} s_{b,j} - H \sum_i (s_{a,i} + s_{b,i})$$

where $\epsilon_{ij} = \epsilon$ if sites i and j are neighbors (each site has γ neighbors of the other type of site) and zero otherwise.

(a) Using the mean field approximation, show that the average spin on the A lattice is given by the transcendental equation

$$\langle s_a \rangle = -\tanh(\beta(\epsilon\gamma[-\tanh(\beta(\epsilon\gamma\langle s_a \rangle - H))] - H))$$

with a similar expression for $\langle s_b \rangle$.

(b) The total magnetic moment is $M = \frac{1}{2} m N(\langle s_a \rangle + \langle s_b \rangle)$ where m is the magnetic moment for a single spin. For $H = 0$, qualitatively describe how M varies with temperature.

3. Consider an anti-ferromagnet lattice (see previous problem). (a) Find the isothermal susceptibility, that is, obtain

$$\chi_T = \left(\frac{\partial M}{\partial H} \right)_T$$

Warning: Stay organized; expressions can get messy.

(b) Show that in the limit of small molecular field (i.e., when $\langle s_a \rangle$ and $\langle s_b \rangle$ are small) that

$$\chi_T \approx \frac{Nm}{k(T + T_c)}$$

where $T_c = \gamma\epsilon/k$. This is Neél's law for antiferromagnets.