Homework 16 (Due Thursday, April 12th)

1. Solve problem 12.4 in Pathria and Beale (pg. 464). I've been using a different notation from the book; in their notation: $L = \langle s \rangle$ without mean field approximation; $\bar{L} = \langle s \rangle$ using the mean field approximation; $\mu B = H$; $qJ = \gamma\epsilon$. Note that q in this last expression, which appears in the exponential, is *not* the q in equation (1) of the problem.

2. Consider a small Ising system with only three distinguishable spin sites (s_1, s_2, s_3), $s_i = +1$ or -1. Each site interacts with all of the other spins; the interaction energy for a pair is $-\epsilon$ when the pair is aligned and $+\epsilon$ when they are anti-aligned ($\epsilon > 0$). The total energy for a site is the sum of the interactions with the other sites; there is no external field.

(a) Find the canonical partition function for this system without using any approximations.

(b) Find the energy of the system as a function of temperature and state how U goes as $T \to 0$ and as $T \to \infty$.

(c) Find the canonical partition function for a similar system of N spins for which each site interacts with *all* other sites. Express your answer in the form,

$$Q_N = \sum_{N_+=0}^{N} f(N_+)$$

where N_+ is the number of sites with spin $+1$.