Homework 17 (Due Tuesday, April 17th)

1. Consider a system with the canonical partition function,

$$Q_N = \frac{1}{N!}\left(\frac{V - Nb}{N\lambda^3}\right)^N \exp(aN^2/kTV)$$

where a and b are constants and λ is the thermal wavelength.
(a) Find the equation of state $P(T, \upsilon)$ where $\upsilon = V/N$ is the volume per particle.
(b) Express the reduced pressure $\overline{P} = (P - P_c)/P_c$ in terms of the order parameter $\eta = \upsilon_c/\upsilon - 1$ and the reduced temperature $\tau = (T - T_c)/T_c$. The critical pressure, temperature, and volume per particle are P_c, T_c, and υ_c, respectively.

2. For the system in the previous problem:
(a) Find the critical exponent α.
(b) Find the critical exponent β
(c) Find the critical exponent γ
(d) Find the critical exponent δ.
In each case show how the result is derived, don't just quote a value.

3. Suppose that the thermodynamic potential (i.e., free energy) near the critical point was given by

$$G(\eta, \tau, F) = C_0 + \frac{C_2}{2}\tau\eta^2 + \frac{C_6}{6}\eta^6 - C_* F\eta$$

where η is the order parameter, F is the external field,$\tau = T/T_c - 1$, and the C's are positive constants. Find the critical exponents β, γ, and δ and confirm the scaling relation $\gamma = \beta(\delta - 1)$.

4. Solve Problem 13.6 in Pathria and Beale and find an expression for the correlation, $\langle s_i s_{i+1}\rangle$. Note that you'll obtain a partition function similar to that given in (13.2.10); although the problem says to solve the problem exactly you do make the approximation that $N \gg 1$.