Homework 3 (Due Tuesday, February 7th)

1. Show that when the coefficient of thermal expansion, α, is of the form, $\alpha = 1/(T + T_0)$, where T_0 is a constant, that $(\partial C_P/\partial P)_T = 0$.

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2. The adiabatic volume expansivity may be defined as

$$\alpha_S = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_S$$

(a) Show that

$$\alpha_S = -\frac{C_V}{VT}\frac{K_T}{\alpha}$$

(b) Find α_S for an ideal gas.

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3. For a given substance, the entropy per particle, $s = S/N$, is,

$$s = s_0 + R\ln\left(\frac{v-b}{v_0-b}\right) + \frac{3R}{2}\ln(\sinh[c(u + a/v)])$$

where $v = V/N$ and $u = U/N$ are the volume and energy per particle, respectively. The parameters R,s_0,b,v_0,a and c are constants.

(a) Find the equation of state for this material, that is, obtain an equation for the pressure in terms of temperature and volume.

(b) Show that the heat capacity per particle at constant volume is,

$$c_V = \frac{3R/2}{1 - (3RcT/2)^2}$$

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4. Consider a system with the equation of state

$$(P + f(V))(V - bN) = NkT$$

where b is a constant.

(a) In general $C_V = C_V(V,T)$; show that for this system C_V is independent of volume.

(b) Given that the isothermal compressibility for this system is

$$K_T = \frac{V - bN}{PV}$$

find $f(V)$ and show that the equation of state may be written as

$$P = \frac{NkT - \mathcal{E}}{V - bN}$$

where $\mathcal{E}$ is a constant.

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5. A given material has an equation of state

$$PV = AT^3$$

where A is a known constant. The internal energy is

$$E = BT^a \ln(V/V_0) + f(T)$$

where B, a, and V_0 are all constants and $f(T)$ is a function that only depends on the temperature. Find B and a. [Hint: Find a relation that relates $(\partial E/\partial V)_T$ to the pressure and match terms.]