Homework 6 (Due Thursday, February 16th)

1. Consider two systems, A and B, with states $n_A = 1, \ldots, \Gamma_A$ and $n_B = 1, \ldots, \Gamma_B$.

(a) We define the entropy of system A as

$$S_A = -k \sum_{n_A} \mathcal{P}_A(n_A) \ln \mathcal{P}_A(n_A)$$

and similarly for system B. Show that if we consider the combined system, A and B together, with the assumption that the interaction between the systems A and B is negligible, that entropy is extensive. Specifically, show that by assuming that $\mathcal{P}_{AB}(n_A, n_B) = \mathcal{P}_A(n_A)\mathcal{P}_B(n_B)$ we find that $S_{AB} = S_A + S_B$.

(b) Suppose that we defined the entropy of system A in a more general form as,

$$S_A = -k \sum_{n_A} \mathcal{P}_A(n_A) f(\mathcal{P}_A(n_A))$$

and similarly for system B. Show that the requirement that entropy is extensive means that $f(x) = \ln x$. Again, assume that the interaction between the systems is negligible.

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2. (a) Use the derivation in Appendix C of Pathria and Beale replacing equation (C.4) by the integral,

$$\int_0^\infty e^{-r} r^2 \, dr = 2$$

and show that,

$$V_{3N}(R) = \int \ldots \int \prod_{i=1}^{N} \left(4\pi r_i^2 dr_i\right) = \frac{(8\pi R^3)^N}{(3N)!}$$

where the integrals are constrained to the region of space such that

$$0 \leq \sum_{i=1}^{N} r_i \leq R$$

(b) Use this result to formulate the entropy, $S(U, V, N)$, for an extremely relativistic ideal gas using the micro-canonical ensemble. Specifically, assume that the gas has N particles and the energy of each particle is $\epsilon = pc$ where c is the speed of light. There is no interaction energy between the particles. You may assume that the number of states with energy $E = U$ is approximately the same as the number with $E \leq U$.

(c) Show that in an adiabatic process for this gas PV^γ is constant with $\gamma = 4/3$.