Projectile motion in perspective

As physicists, drawings of projectile trajectories always catch our eye. My favourites are the illustrations that appear in golfing magazines and books showing the path of the ball from tee to green. That is, the golfer is in the foreground, near the viewer, while the ball lands in the far distance. In these drawings the trajectory is not symmetric about the midpoint (point of maximum height) because, due to perspective, lengths appear shorter when they are farther away. It is well known that aerodynamic lift and drag play a significant role in the flight of a golf ball; however, suppose that the only force on the ball was constant downward gravity (e.g., golfing on the moon). The question we will consider is: how does one draw the parabolic arc of projectile motion in perspective? After answering this question I will mention a few ideas on how to use these drawings in the classroom.

To start, take a look at figure 1, which shows projectile motion without perspective. Consider the rectangle formed by the line between the start and end points, the horizontal line at the maximum height, and the vertical lines from the start and end points up to the maximum height line. To sketch in the parabolic arc, we may draw three evenly spaced vertical lines and three evenly spaced horizontal

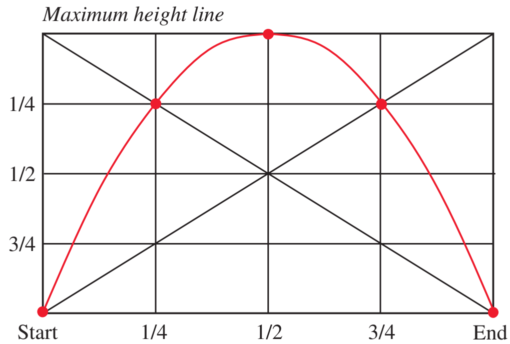

Figure 1. *Projectile motion without perspective.*

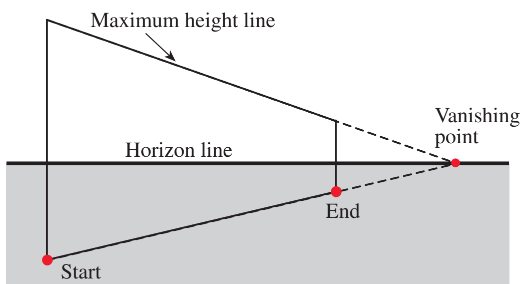

Figure 2. *Rectangular plane of the trajectory, in perspective.*

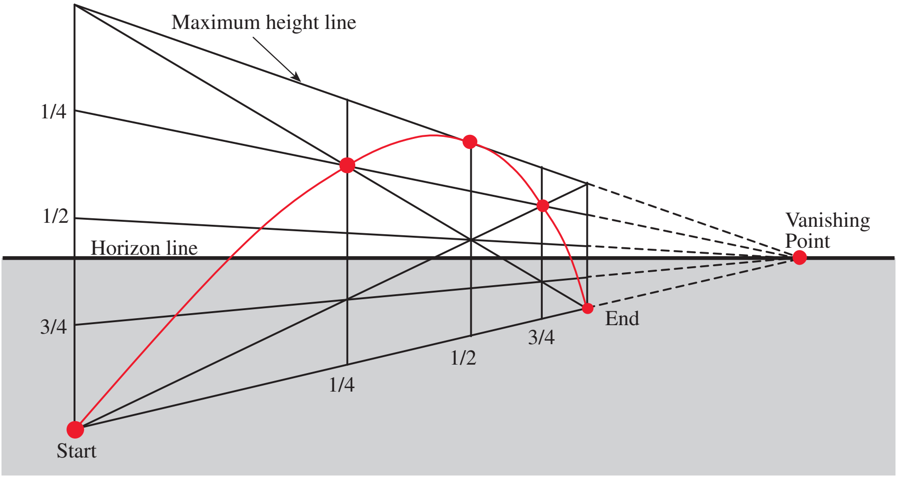

Figure 3. *Projectile motion, in perspective.*

lines. Note that the diagonals of the rectangle cross the intersections of these horizontal and vertical lines. The top of the parabolic arc passes through the centre point of the maximum height line. The arc also passes through points one quarter of the distance from the maximum height line that are 1/4 and 3/4 of the horizontal distance between the start and end points. If desired, we could further subdivide the rectangle to locate more points on the arc (e.g., at 1/8 of the horizontal distance the arc is 9/16 from the maximum height line).

We will now transcribe the parabolic trajectory of figure 1 into a perspective drawing. To start, draw a horizontal line representing the horizon; this horizon line is at eye-level for the viewer looking horizontally into the distance. Select the location of the start and end points, which we take as being on level ground. Draw a line from the start point, passing through the end point, and extending out to the horizon line (see figure 2). The point where this line meets the horizon is the vanishing point. All lines parallel to the start–end line pass through this vanishing point.[1]

Next, we construct the rectangle of figure 1, which we imagine as a giant billboard on which we are going to draw the trajectory. First, choose the point where the maximum height line is directly above the starting point of the trajectory and draw the left edge of the rectangle. Draw a line from the top of this line to the vanishing point; this is the maximum height line seen in perspective. Complete the rectangle by drawing a vertical line from the end point of the trajectory up to the maximum height line (see figure 2).

The next task will be to partition the large rectangle into 16 equal rectangles. First subdivide the left edge of the rectangle into four equal parts. Draw lines from each of these increments out to the vanishing point; these are the horizontal lines of the rectangle in perspective (see figure 3). Now draw the diagonals of the rectangle, that is, straight lines between opposite corners. Recall that the diagonals cross the horizontal lines at the intersections with the vertical lines so draw the four vertical lines such that they pass through those crossings (compare figures 1 and 3).

Finally, the points of the arc indicated in figure 1 are transcribed by locating the corresponding corners of the small rectangles (see figure 3). As mentioned earlier, more points of the arc can be located by subdividing the large rectangle into finer divisions. Also, the entire drawing can be executed with just a straight edge and a triangle (i.e., without a ruler) since a rectangle is bisected at the intersection of its diagonals.

Projectile motion is more complicated when air resistance is significant and one needs to resort to the computer to obtain the trajectory [1]. By the technique described above, one can draw a Cartesian grid in perspective and plot the numerically

[1] In perspective, straight, parallel railroad tracks extending to the horizon converge and meet at the vanishing point.

computed points. The shape of the trajectory is asymmetric, with the point of maximum height closer to the end point of the trajectory. Because of this asymmetry, when drawn in perspective the trajectory looks different depending on whether the start point or the end point is in the foreground. Interestingly, in the latter case the trajectory, in perspective, looks more like a symmetric parabola.

There are a variety of ways you can use perspective drawing in the classroom. You can show students how to construct these drawings and then have them create their own; each drawing will look different depending on the perspective. Have students locate and sketch in some of the other points of the trajectory. Collect drawings of projectile motion from sports magazines and ask students to determine whether or not the trajectory is a parabolic arc. Assuming that the spin of the golf ball produces a constant horizontal force on the ball, ask students to draw a sliced or hooked golf shot in perspective (or a curve ball as seen by a baseball catcher). As a project, students could write a computer graphics program [2] that would calculate and draw trajectories in perspective (such a program is available from the author, upon request, to instructors).

Drawing is a also good way to present kinematics to non-science majors. Recently I gave a guest lecture on the physics of simple motion in the School of Art and Design at San José State University. Graphic arts majors study perspective in the context of static objects, but learning to draw motion realistically is useful for animators and illustrators.

Physics instructors often complain that students eagerly memorize the formulas of kinematics yet seem unable to visualize the qualitative form of the motion. So if your students are going to be doodling in class, maybe you can get them to think about what they are drawing!

The author wishes to thank John Clapp for teaching him perspective drawing.

References

[1] Garcia A L 2000 *Numerical Methods for Physics* 2nd edn (Upper Saddle River, NJ: Prentice Hall) ch 2

[2] For a mathematical treatment of perspective, see Hearn D and Baker M P 1997 *Computer Graphics* 2nd edn (Upper Saddle River, NJ: Prentice Hall)

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