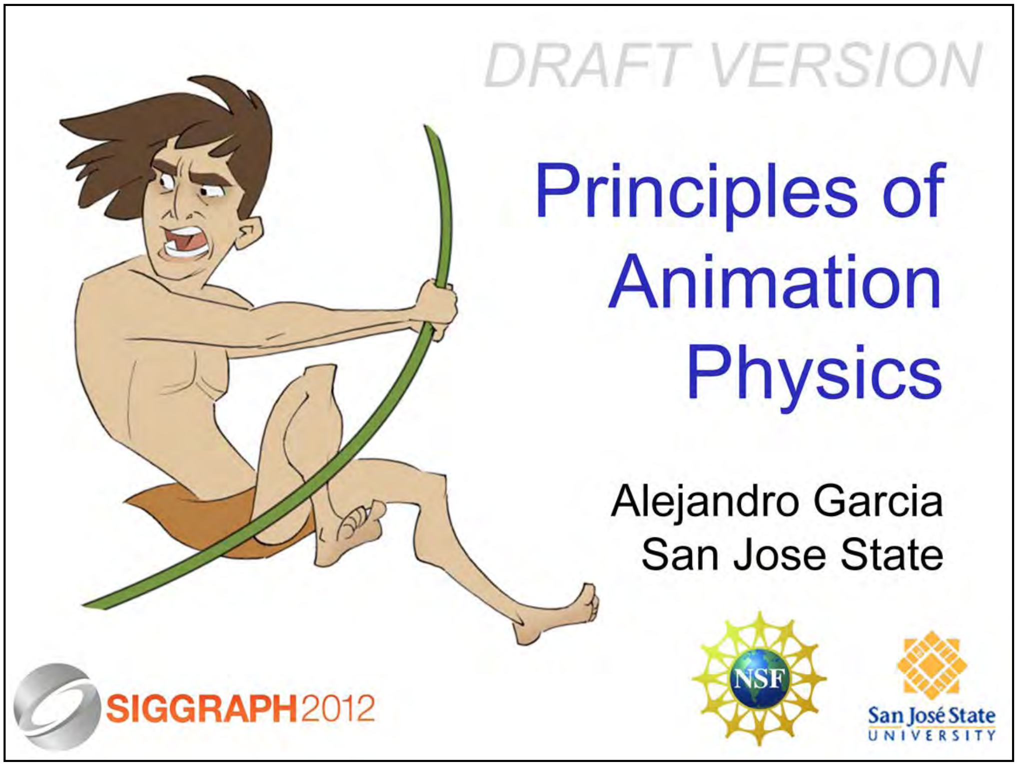
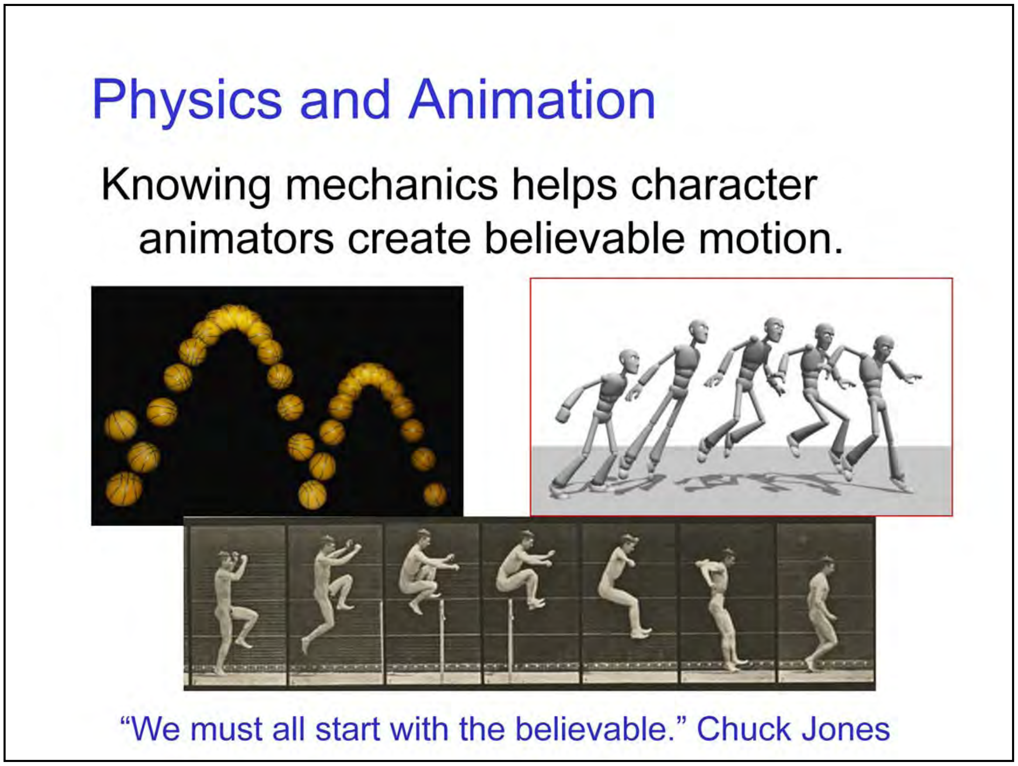


Acknowledgements: The *Physics for Animation Artists* project has been supported by the National Science Foundation's Transforming Undergraduate Education in Science, Technology, Engineering and Mathematics program. The author also thanks his colleagues at San Jose State University and Dreamworks Animation.

Biosketch: Professor Alejandro Garcia has taught *Physics of Animation* at San Jose State since 2009. In 2011 he spent a one-year professional leave as physicist-in-residence at Dreamworks Animation SKG. During his time with Dreamworks he worked in the Division of Artistic Development, presenting over 30 classes and special lectures for various studio departments and consulted on *Madagascar 3: Europe's Most Wanted*. This work was highlighted on Science Nation; see: http://tinyurl.com/8ynw2tt. Dr. Garcia is also the author of the textbook, *Numerical Methods for Physics*, and has published over 80 professional articles in the fields of computational physics, statistical mechanics, and fluid mechanics.

lllustration by Charlene Fleming



Why should a character animator care about physics?

If you're going to be a surgeon then you need to know advanced anatomy; for life drawing a basic knowledge of muscles and bones is helpful. If you're going to be an engineer then you need to know calculus and physics; as a character animator a basic understanding of mechanics and bio-mechanics is helpful. In *Chuck Amuck*, Chuck Jones writes, “Comparative anatomy is a vital tool of the complete animator or director.” The purpose of this course is to make physics another tool in your animator's toolbox.

At San Jose State I teach *Physics of Animation*, a one semester course for animation artists. The first eight weeks of the semester cover mechanics and bio-mechanics; this SIGGRAPH course is a distillation of those 16 lectures down to a three hours. The focus of today's course is character animation, though many of the concepts are also useful for character effects, such as hair and clothing. Physics is also useful for effects animators and for lighting artists but we won't have time to get to those areas today.

Bouncing Ball: http://commons.wikimedia.org/wiki/File:Bouncing_ball_strobe_edit.jpg

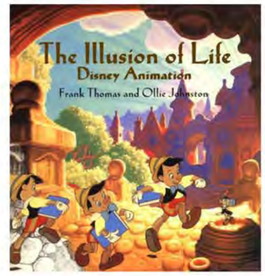
Character Jumping by Corey Tom

Man Jumping:
http://commons.wikimedia.org/wiki/File:Animal_locomotion._Plate_154_(Boston_Public_

Library).jpg

Principles of Animation

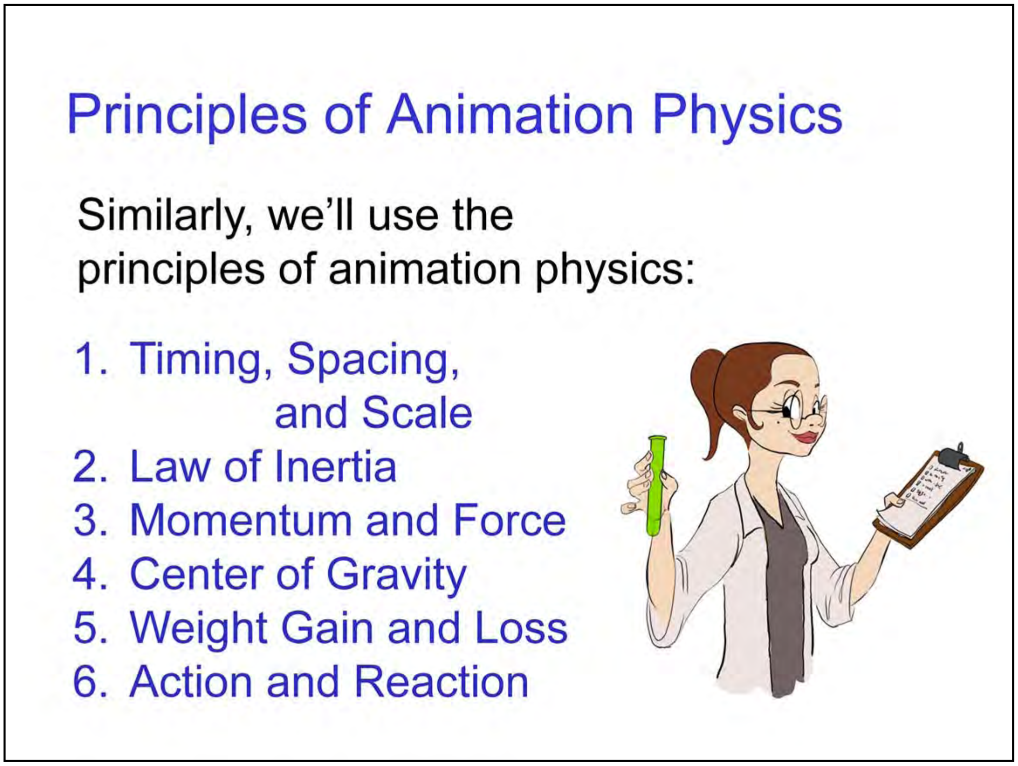
In *The Illusion of Life*, Frank Thomas and Ollie Johnston list a set of basic principles for animation.



1. Squash & Stretch
2. Timing
3. Anticipation
4. Staging
5. Follow Through & Overlapping Action
6. Straight Ahead & Pose-to-Pose Action
7. Slow In and Slow Out
8. Arcs
9. Exaggeration
10. Secondary Action
11. Appeal

All animators are familiar with the “Principles of Animation” as described by Ollie Johnston and Frank Thomas in their book *The Illusion of Life: Disney Animation*. These principles, such as “Squash and stretch” and “Anticipation”', give helpful guidance to animators. And since they describe motion it's not surprising that many of the principles of animation are based on physical laws. For example, the animation principle of follow-through is based on the Law of Inertia.

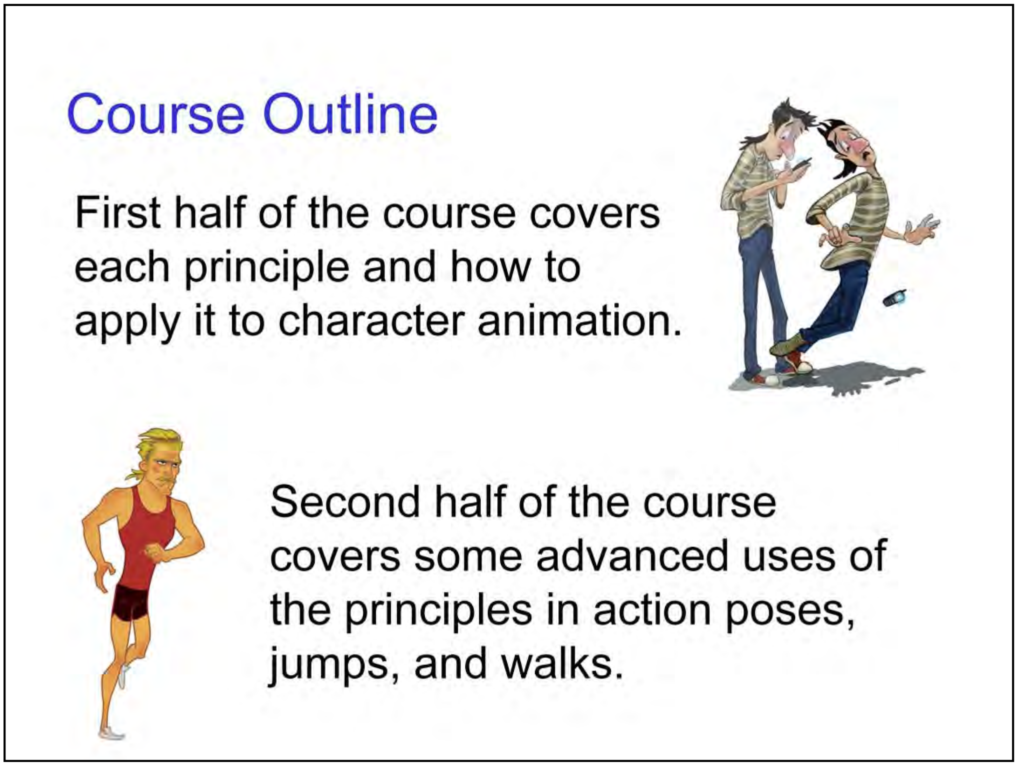
Book Cover: http://en.wikipedia.org/wiki/File:Book_the_illusion_of_life.jpg



We'll follow the successful model introduced by Johnston and Thomas and make a list of principles to organize our understanding of physical motion. Specifically, our Principles of Animation Physics will be:

1. Timing, Spacing, and Scale
2. Law of Inertia
3. Momentum and Force
4. Center of Gravity
5. Weight Gain and Loss
6. Action-Reaction

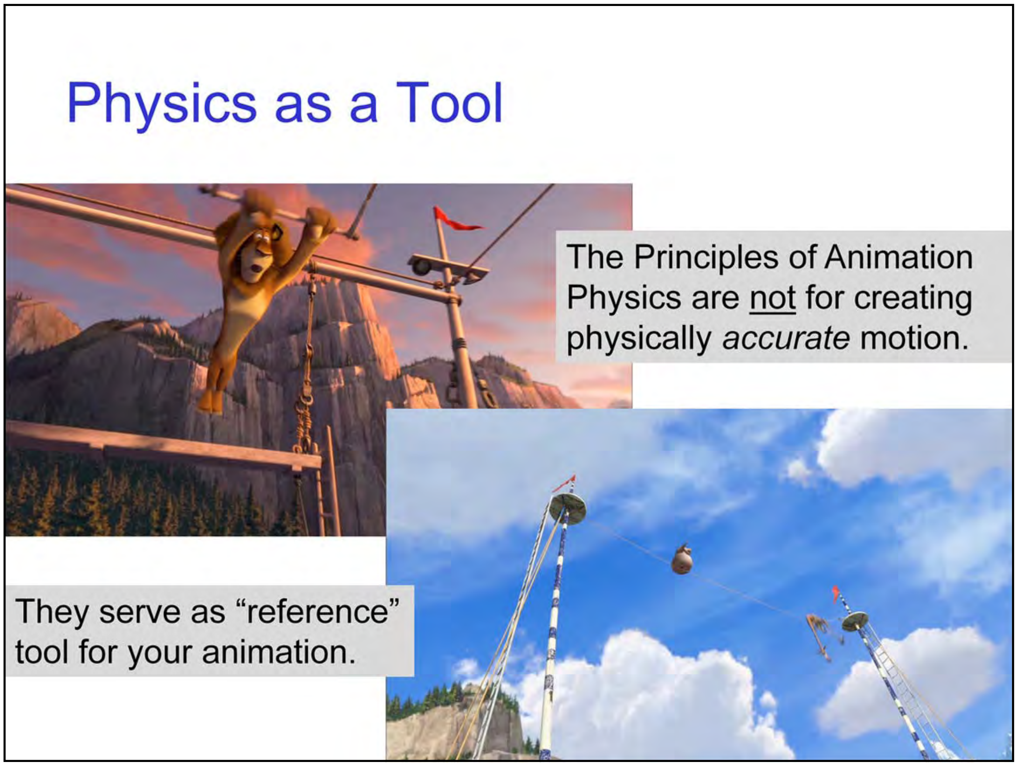
Illustration by Rebbaz Royee



In the first half of the course I'll explain each principle and how it applies in simple situations. Then in the second half we'll use these principles to break down and understand such actions as jumping and walking.

Falling man illustration by Rebbaz Royee

Running man illustration by Charlene Fleming

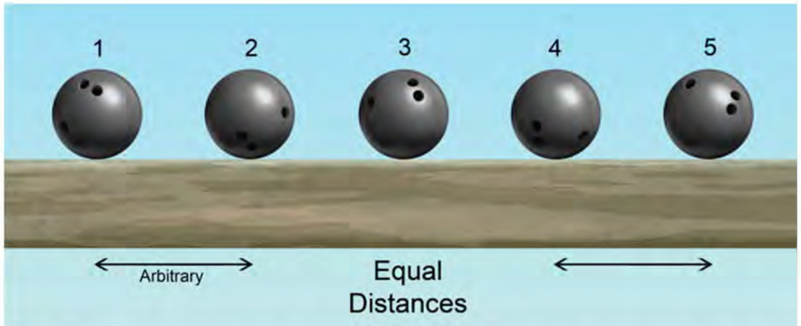


One last thing before we start: These principles of animation physics are <u>not</u> for the purpose of creating physically accurate motion. They are guides to help you interpret and understand physical motion. Animators commonly use video reference, not to copy it but to extract the essence that they need. Using physics you'll be able to imagine scenarios, such as a giraffe on a tightrope or a lion on trapeze, that you'll never find on YouTube. To quote Chuck Jones once more, "We must all start with the believable. That is the essence of our craft."

Part I: Principles of Animation Physics

#1: Timing, Spacing, and Scale

Uniform Motion



In uniform motion, the velocity is constant so the spacing from frame to frame is constant.

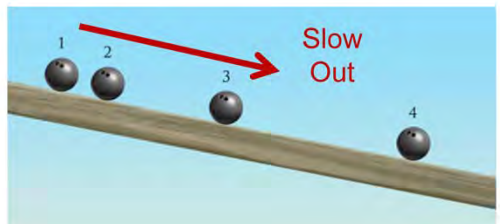
Larger the spacing, the faster the ball is moving.

Uniform motion has the simplest spacing; the velocity is constant so the spacing is constant. The number of frames between each key pose gives the timing.

Illustration by Charlene Fleming

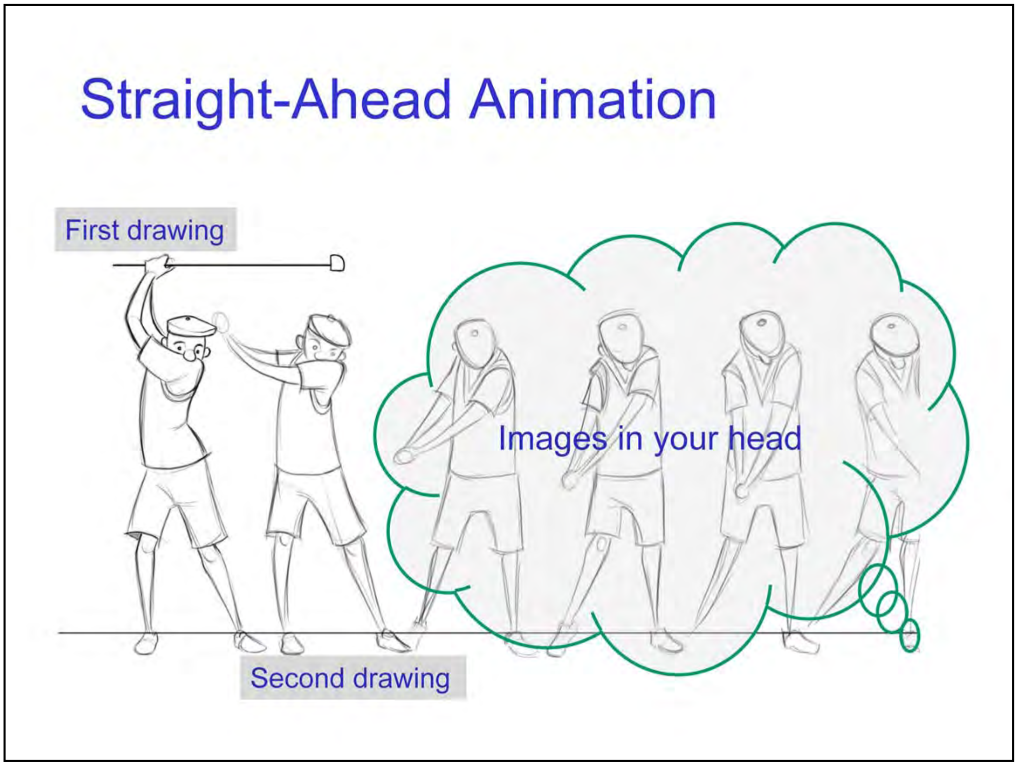
Slow In and Slow Out

If the motion is *not* uniform, the object is either speeding up, slowing down, or changing direction.



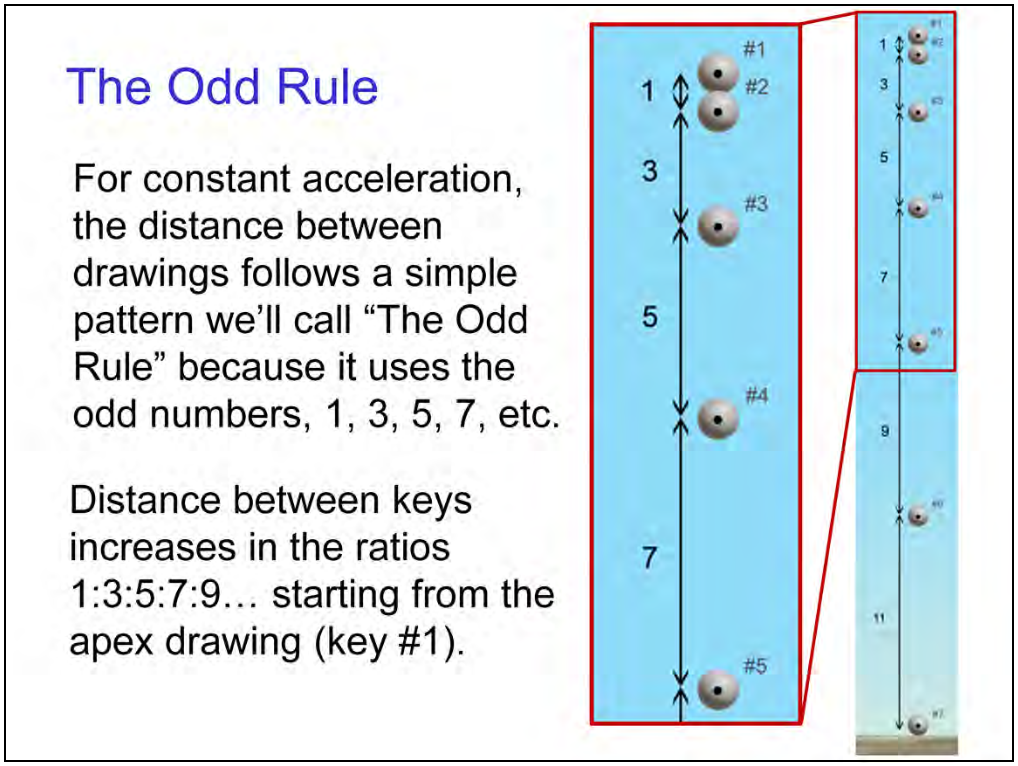
When the motion is not uniform then the spacings are not constant. For example, the character could be moving but losing speed and so it “slows in” to the final pose. Similarly, an object can slow out from an initial pose, with the spacings getting larger as it gains speed. Finally, moving in a new direction is another example of motion that is not uniform. Later we’ll see that in all of these cases there must be forces causing the change in the motion. By the way, “Slow In and Slow Out” is one of Thomas and Johnston’s *Principles of Animation*.

Illustrations by Charlene Fleming



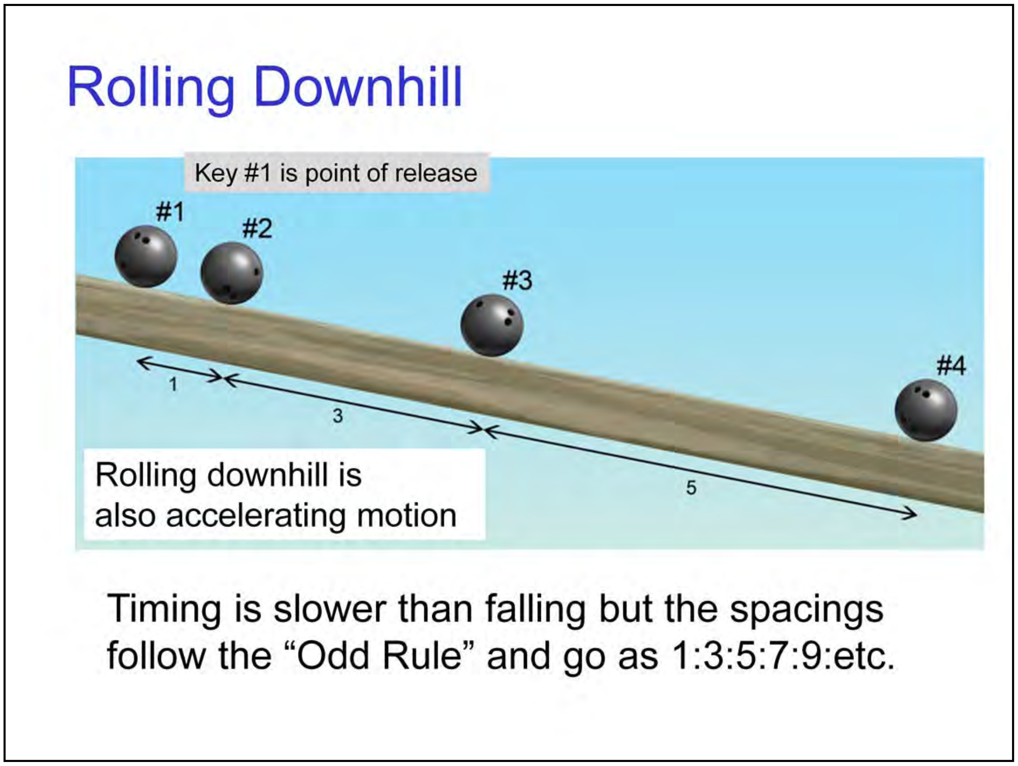
In the “Straight-Ahead” style of animation the animator starts with the first pose, then creates the next pose, then the next, until the action is completed. This style is used in stop-motion animation and sometimes in traditional hand-drawn animation. “Straight Ahead & Pose-to-Pose Action” is a Principle of Animation; Pose-to-Pose Action is described later.

Illustration by Dora Roychoudhury



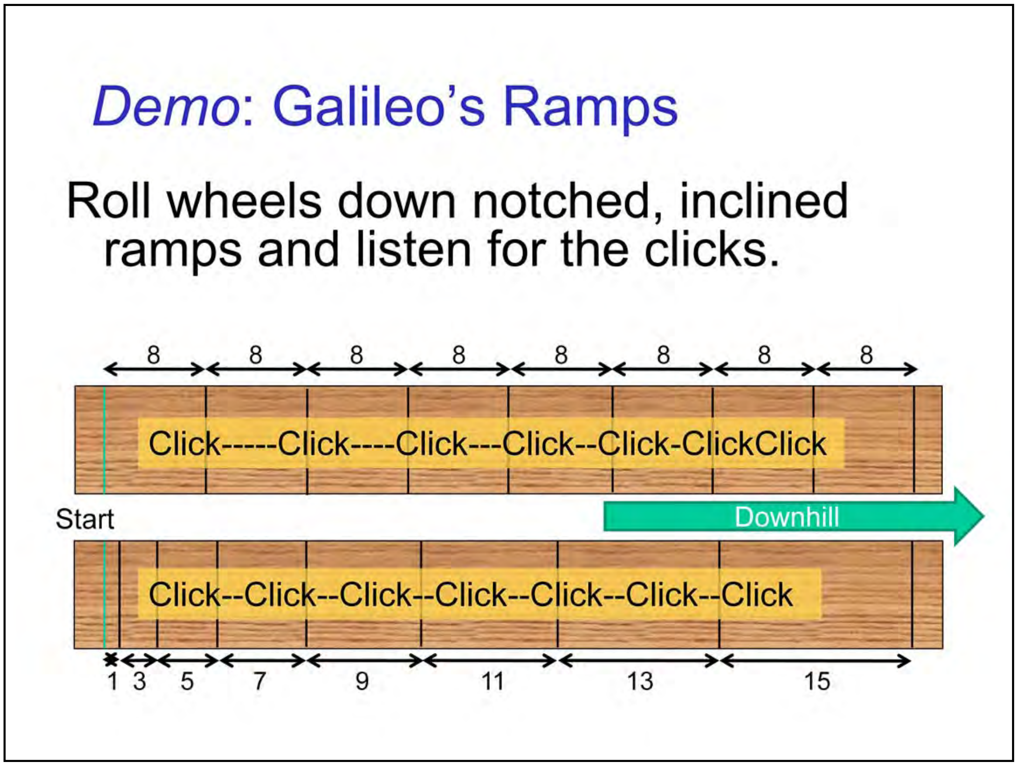
The first exercise that animators practice on is usually the bouncing ball, learning how to adjust the timing and spacing so that it slows into and out of the apex in a believable way. Timing and spacing indicate speed and when spacings change there's acceleration. Because the acceleration of gravity is constant the ball's spacings follow a simple pattern. I call this pattern the "Odd Rule" because the spacings from the apex go as 1, 3, 5, 7, 9, and so on.

Illustration by Charlene Fleming



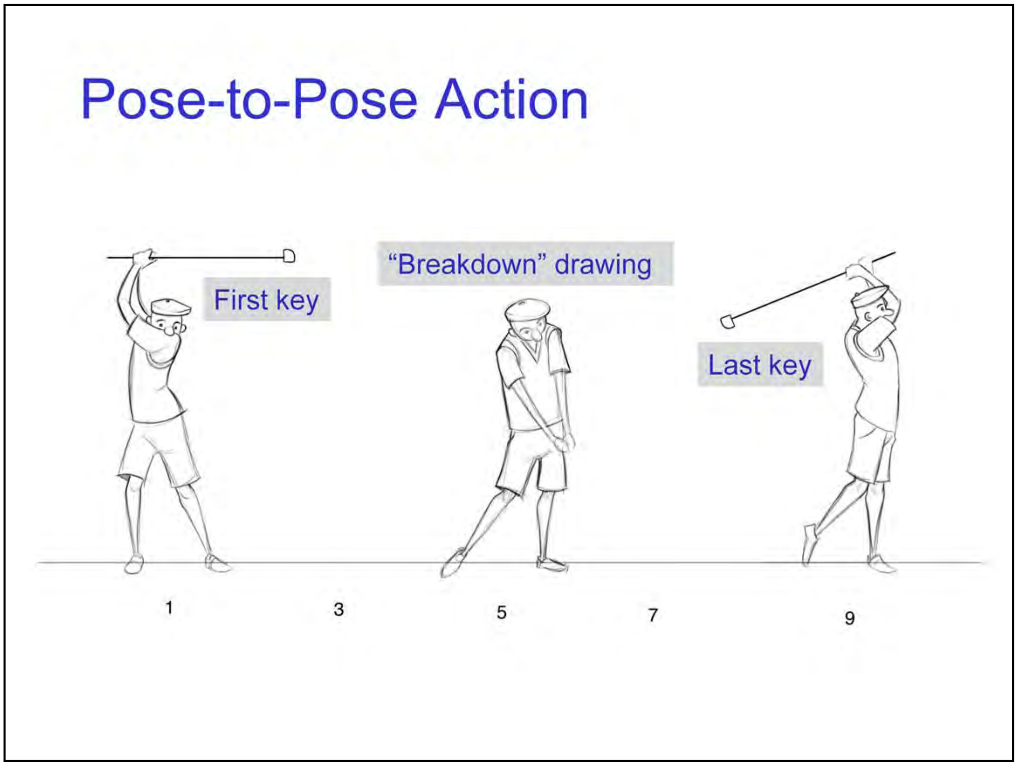
A ball rolling downhill is another example of constant acceleration. Although the timing is slower than falling straight down the spacings still follow the Odd Rule. This pattern isn't a simplified rule-of-thumb; the Odd Rule is as mathematically precise as $x = \frac{1}{2} a t^2$.

Illustration by Charlene Fleming



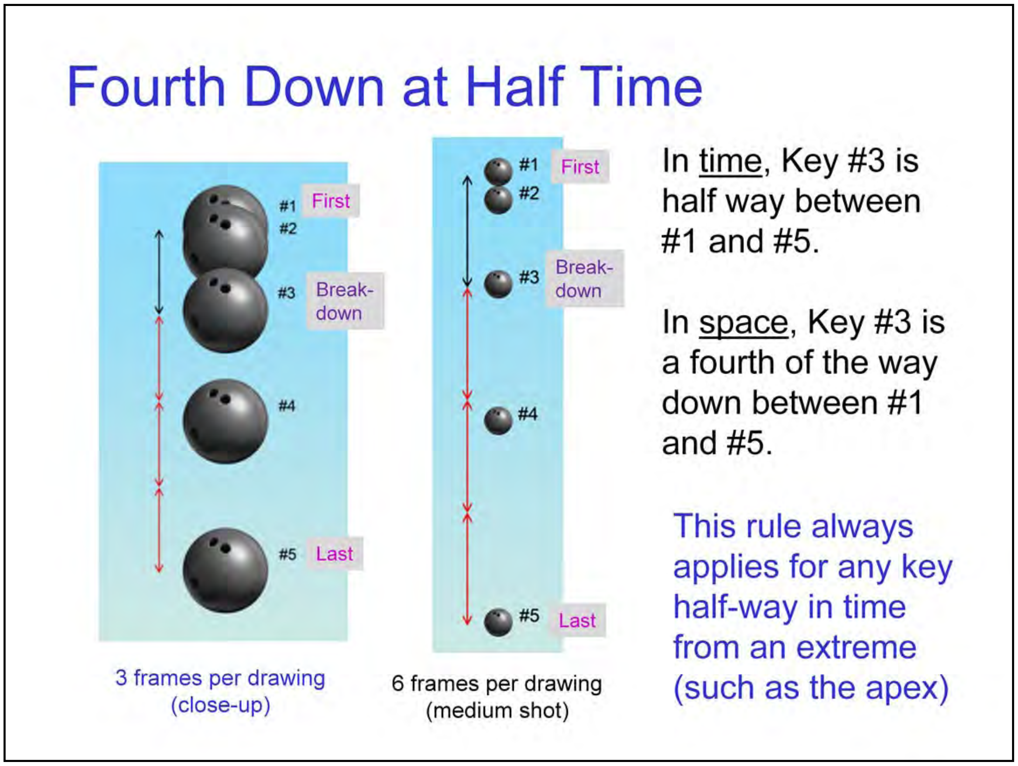
In this demonstration a heavy wheel rolls down ramps that have small notches; when the wheel passes a notch it makes a clicking sound. When the wheel goes down the ramp with evenly spaced notches we hear the clicks coming faster and faster. However, when the wheel goes down the ramp with notches spaced according to the Odd Rule the clicks have a constant beat, telling us that it takes the same time to go from notch to notch.

If the ramp is steep the timing is quick; if the slope is small then the timing is slow. But in all cases the clicks are regular so the spacings are the same, regardless of the timing. This demonstration is based on an experiment described by Galileo in his book, *Two New Sciences*.



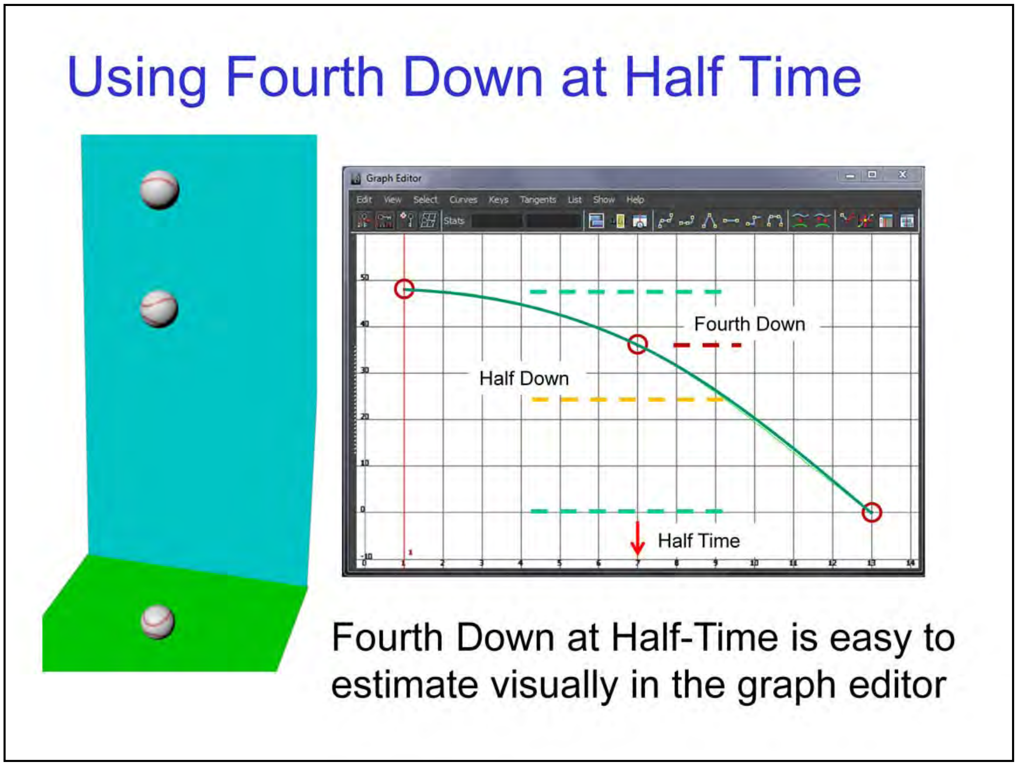
In the "Pose-to-Pose Action" style of animation the animator starts with the first pose and last pose, then the breakdown drawing that's between them, then continues adding in-betweens, until the action is completed. This style is commonly used in CG animation, with the computer interpolating to create the in-betweens.

Illustration by Dora Roychoudhury



The Odd Rule is helpful for animating straight ahead action but for pose-to-pose animation there's a useful corollary that I call "Fourth Down at Half Time." (This mnemonic makes more sense if you know American football.) The rule is very simple: the position of the breakdown pose, between the apex and the final pose, is a quarter of the distance from the apex. As with the Odd Rule, Fourth Down at Half Time applies whenever acceleration is constant, that is, when forces are constant.

Illustration by Charlene Fleming



Fourth Down at Halt Time is easy to use in the graph editor since the position of the breakdown drawing can be quickly estimated visually. The resulting curve is a parabolic arc (note that at the apex the curve’s tangent should be horizontal).

Distance Fallen from an Apex

The formula to compute this table is:

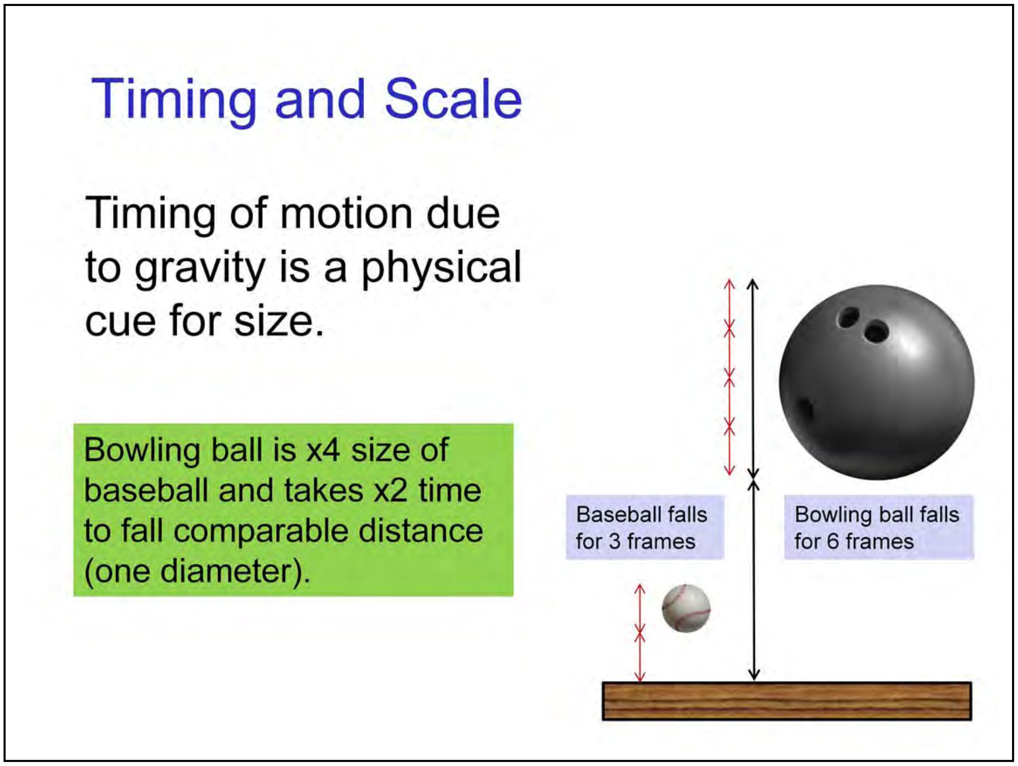
(Distance in inches) =
(1/3 inch) x
(Number of Frames) x
(Number of Frames)

Falling on “twos”

Time (seconds)	Frames	Distance fallen from apex
$^{1}/_{24}$	1	$^{1}/_{3}$ inch
$^{1}/_{12}$	2	1 $^{1}/_{3}$ inches
$^{1}/_{8}$	3	3 inches
¼	6	1 foot
½	12	4 feet
$^{2}/_{3}$	16	7 feet
¾	18	9 feet
1	24	16 feet

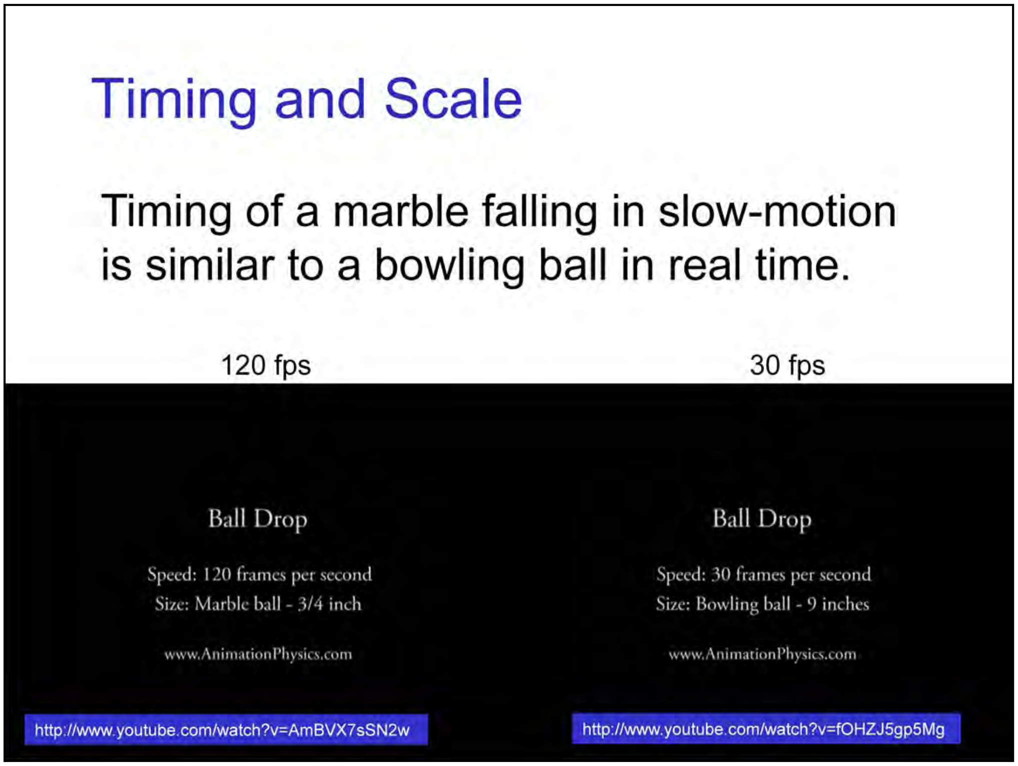
For acceleration under gravity the distance fallen from the apex equals (1/3 inch) times the square of the number of frames. (This is for 24 frames per second, as used in film; note that video typically uses 30 fps.) For example, for 6 frames the distance fallen is (1/3) x (6)x(6) = 12 inches or one foot; for 12 frames the distance fallen is (1/3) x (12)x(12) = 48 inches or four feet.

Illustration by Rebbaz Royee

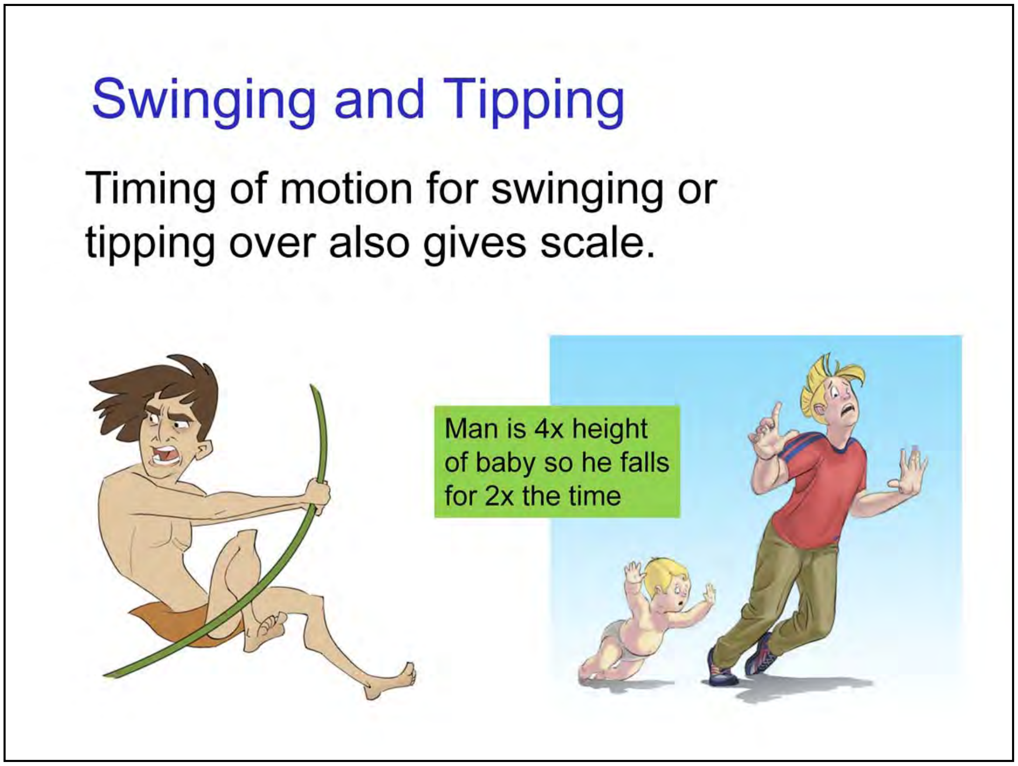


These numerical calculations are rarely useful to character animators but I'm mentioning them because of the importance of timing in creating scale. Suppose that a character jumps off the roof of a house and lands on the ground in 12 frames. The character will feel small because this timing indicates that the house is four feet tall.

Illustration by Charlene Fleming



This connection between timing and scale is used in live-action movies when scale models are filmed with high-speed cameras; by filming at twice the normal speed the models look four times bigger when the action is played back at normal speed. In these videos a marble ball falling in slow motion has a similar timing to a bowling ball. It's important to remember that in order to “stay on model‘” the timing of a character's action must be consistent with the character's size. Just a 20% error in the timing will make a 6 foot character seem like it's only about 4 feet tall.



The connection between timing and scale applies not just to falling motion but also swinging and tipping over. The same goes for other character actions such as walking, running, jumping, etc. since timing of the arms and legs has a similar scaling.

Tarzan illustration by Charlene Fleming

Baby and man illustration by Rebbaz Royee

It’s useful to use the connection between timing and scale to shoot video reference for certain scenes. For example, a two foot chain will swing back and forth like a 72 foot chain when filmed with a high-speed camera recording x6 the normal speed and played back at normally; equivalently, you can use a camera that records at x3 the normal speed and play the video back at half-speed. In both cases the motion of the swinging chain will be slow and lumbering, as if it was 36 times bigger.

Part I: Principles of Animation Physics

#2: Law of Inertia

Newton's Laws of Forces

Newton established three basic laws to explain how motion is caused by forces:

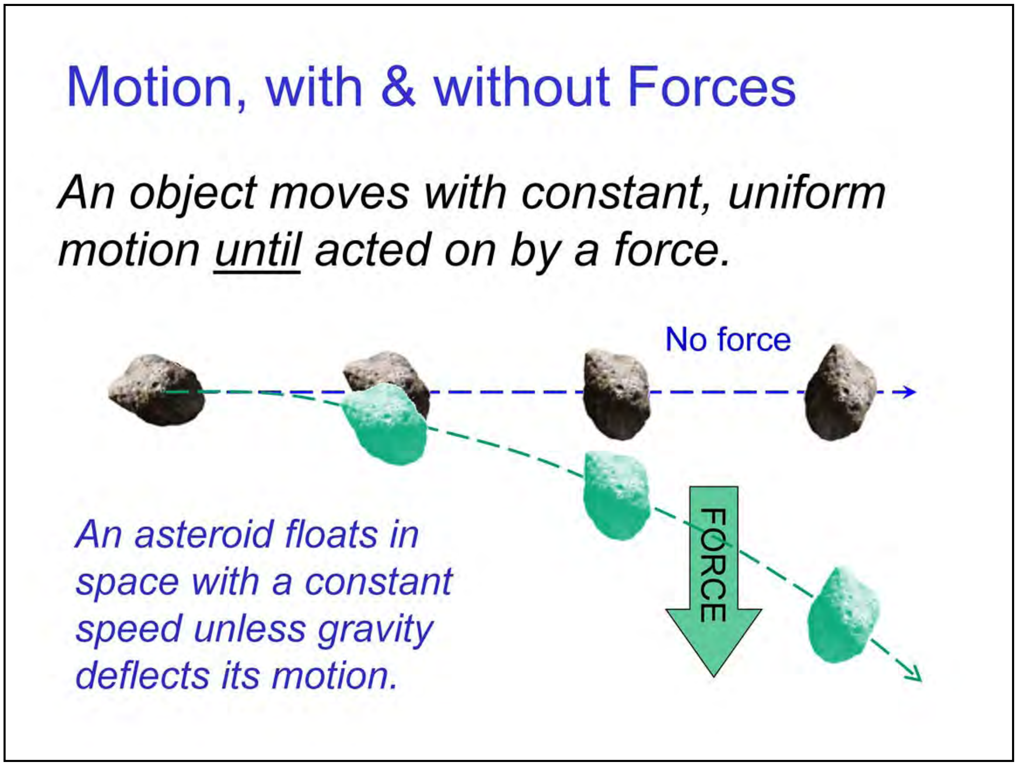
- Law of Inertia
- Momentum and Force (F=ma)
- Action-Reaction Principle

Sir Isaac Newton

Disney and other early animators rediscovered these laws of forces in their studies of motion.

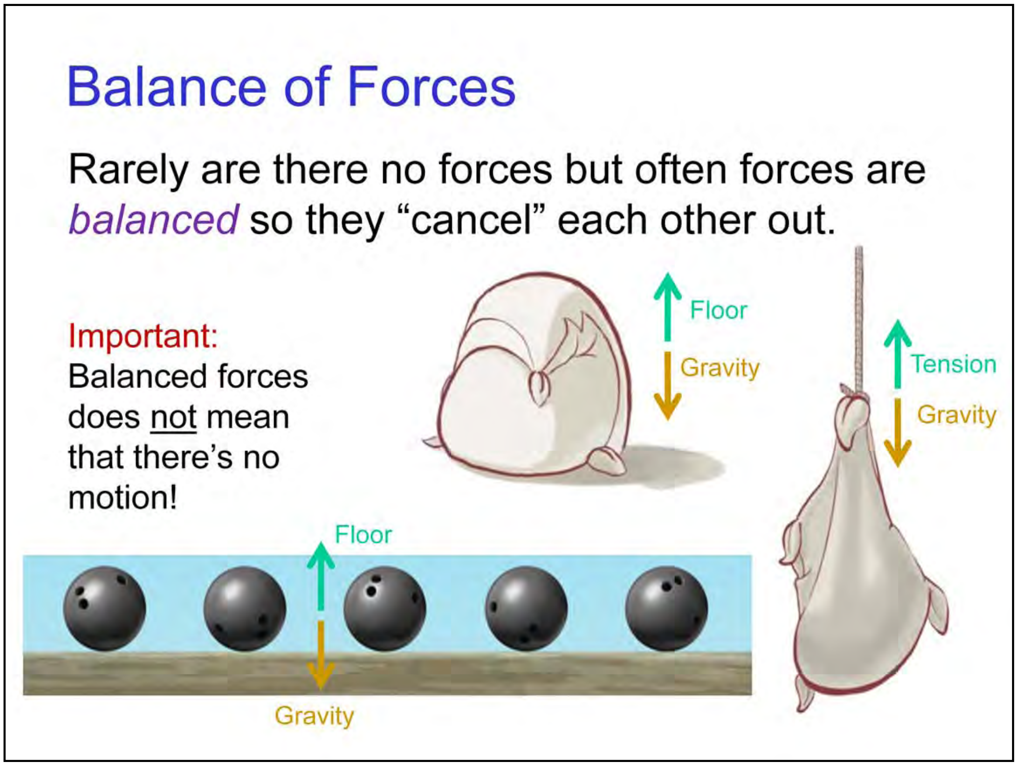
Timing and spacing specify how a character moves but to understand why things move the way they do you need to understand forces. Newton established three basic laws of motion regarding forces and the next five Principles of Animation Physics are based on these laws.

Image of Newton: http://en.wikipedia.org/wiki/File:GodfreyKneller-IsaacNewton-1689.jpg



The Law of Inertia, also known as Newton's First Law of Motion, says that a character will move with constant velocity unless acted on by an unbalanced force. First, let's be clear what's meant by an “unbalanced force.” The simplest form of the Law of Inertia is that an object in motion remains in motion unless acted on by a force, for example, an asteroid in deep space moves with constant velocity. But unless you animated a shot in *Wall-E* it's unlikely you've encounter a situation where there's absolutely no forces.

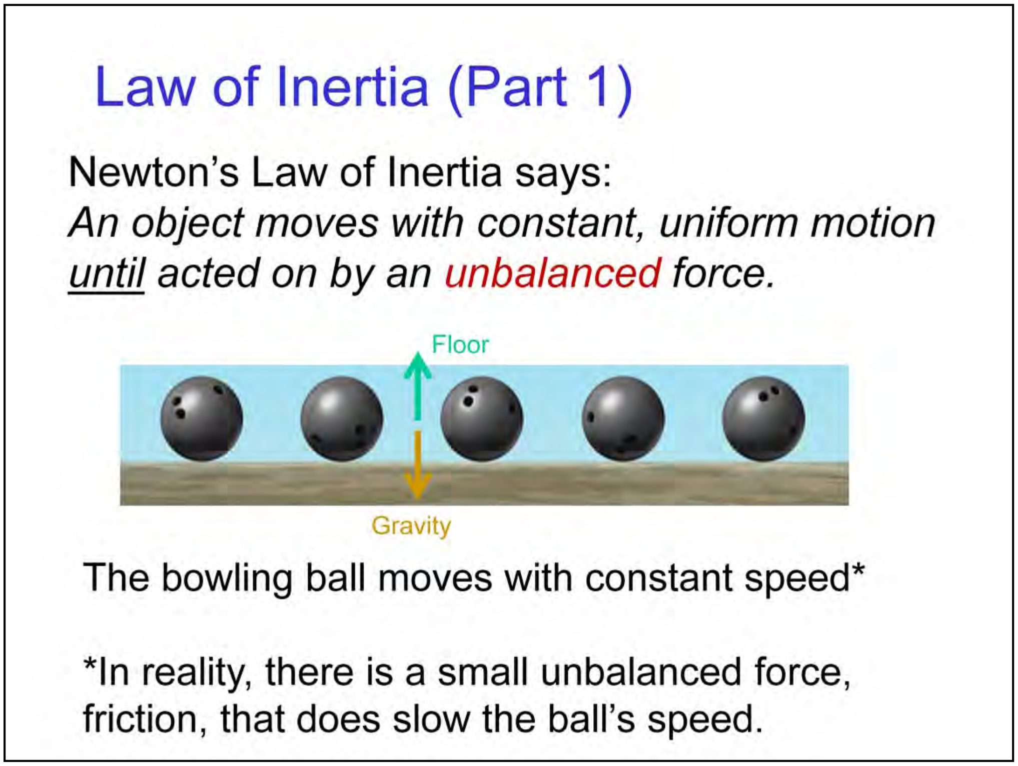
Illustration by Rebbaz Royee



A sack sitting on the floor has two forces on it: gravity pulling downward and the floor pushing upward. These two forces balance each other. A bowling ball rolling on a smooth floor has a similar pair of balanced forces (gravity and the floor). If there's no other force on the bowling ball then there's no unbalanced forces so it moves with constant velocity. Same with the sack, it's speed just happens to be zero.

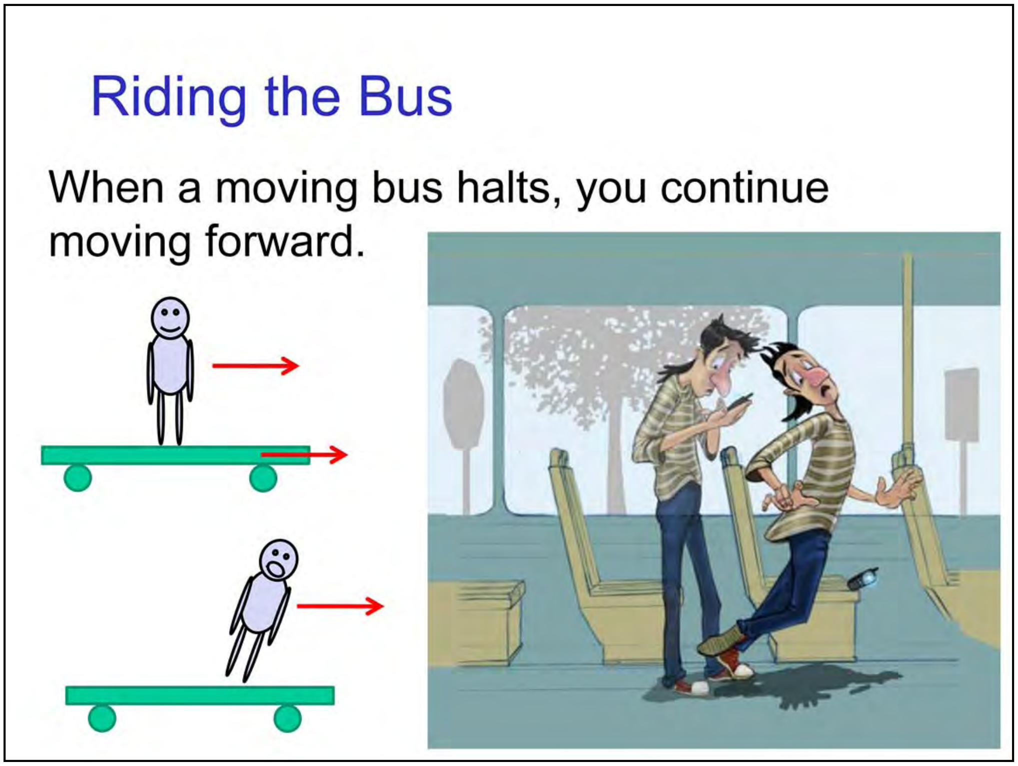
Bowling ball illustration by Charlene Fleming

Sack illustrations by Rebbaz Royee



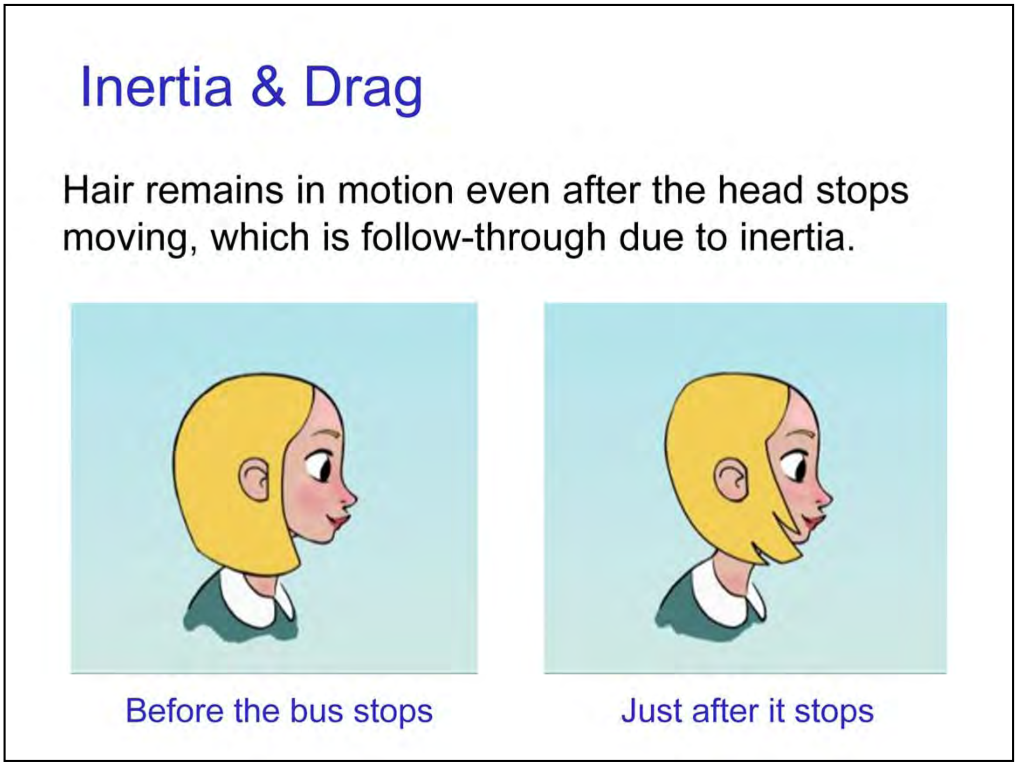
Newton’s Law of Inertia says: An object moves with constant, uniform motion until acted on by an *unbalanced* force. In the example of the bowling ball, it rolls with constant speed since the downward force of gravity is balanced by the upward force of the floor. If there was a third force, say friction or air resistance, that wasn’t balanced then the speed wouldn’t stay constant; the ball would start slowing down.

Illustration by Charlene Fleming



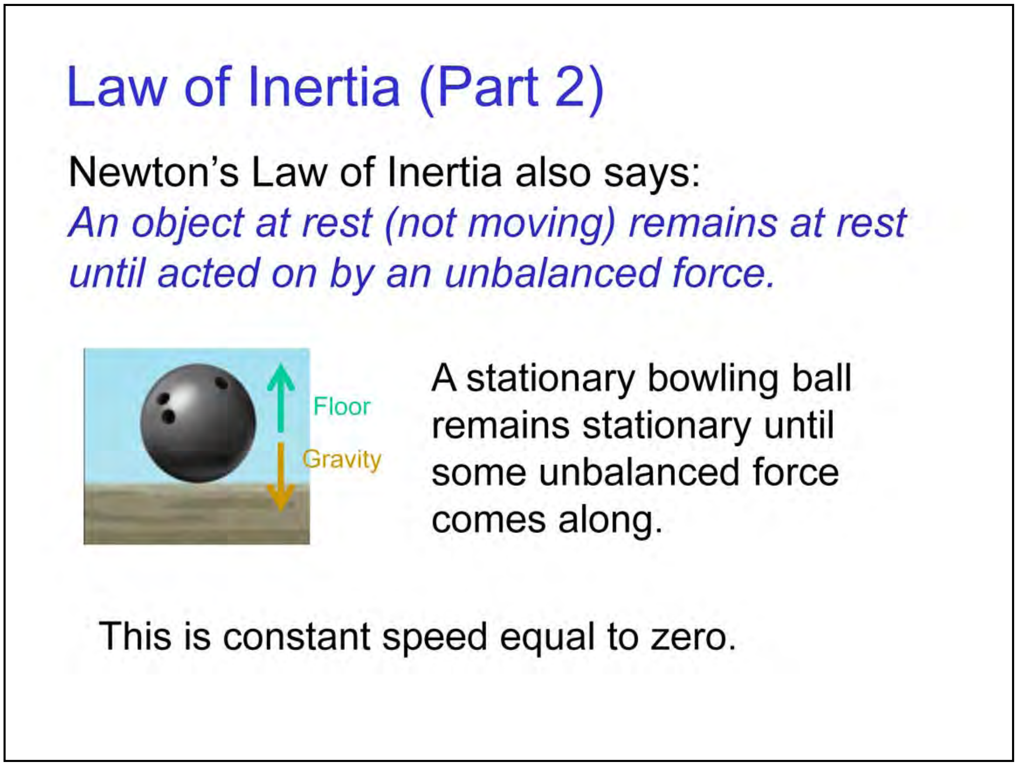
At first this all seems purely academic until you realize that follow-through in animated motion is entirely due to the Law of Inertia. Let's take a simple example of a character standing on a bus. When the bus suddenly stops the character goes flying forward. Before the bus hit the brakes he was moving and by the Law of Inertia, he'll continue moving until a force acts to stop him (such as when he hits the floor).

Illustration by Rebbaz Royee



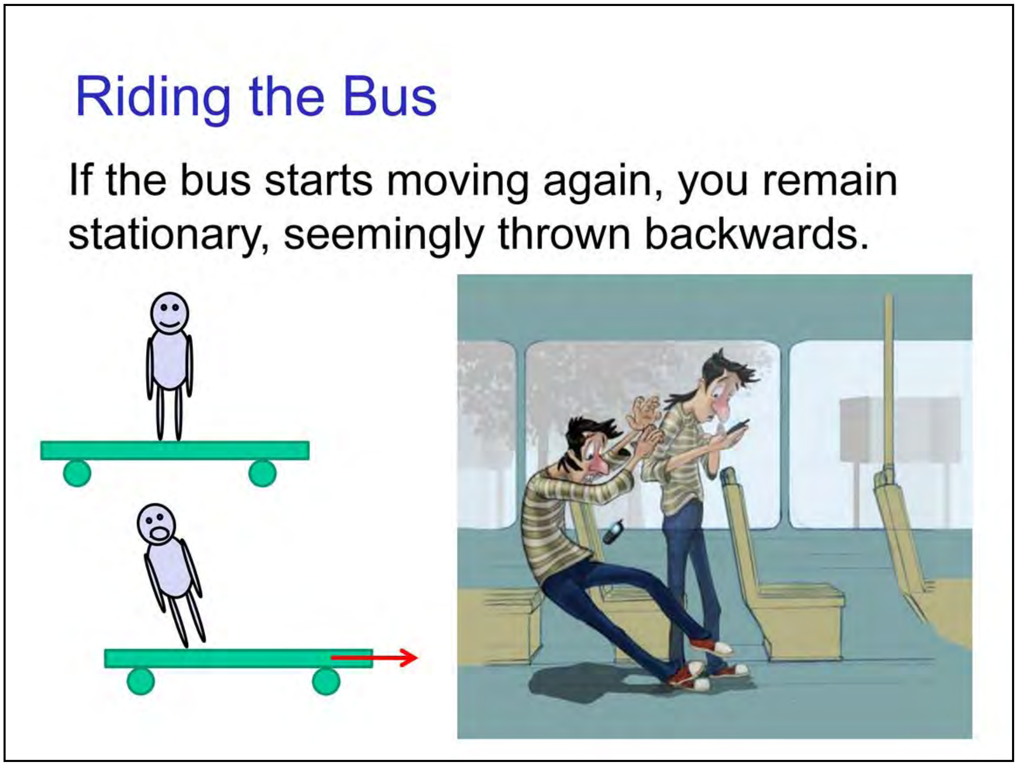
Another example is seen in the follow-through (forward motion) of a seated passenger's hair when the bus hits the brakes.

Illustration by Charlene Fleming



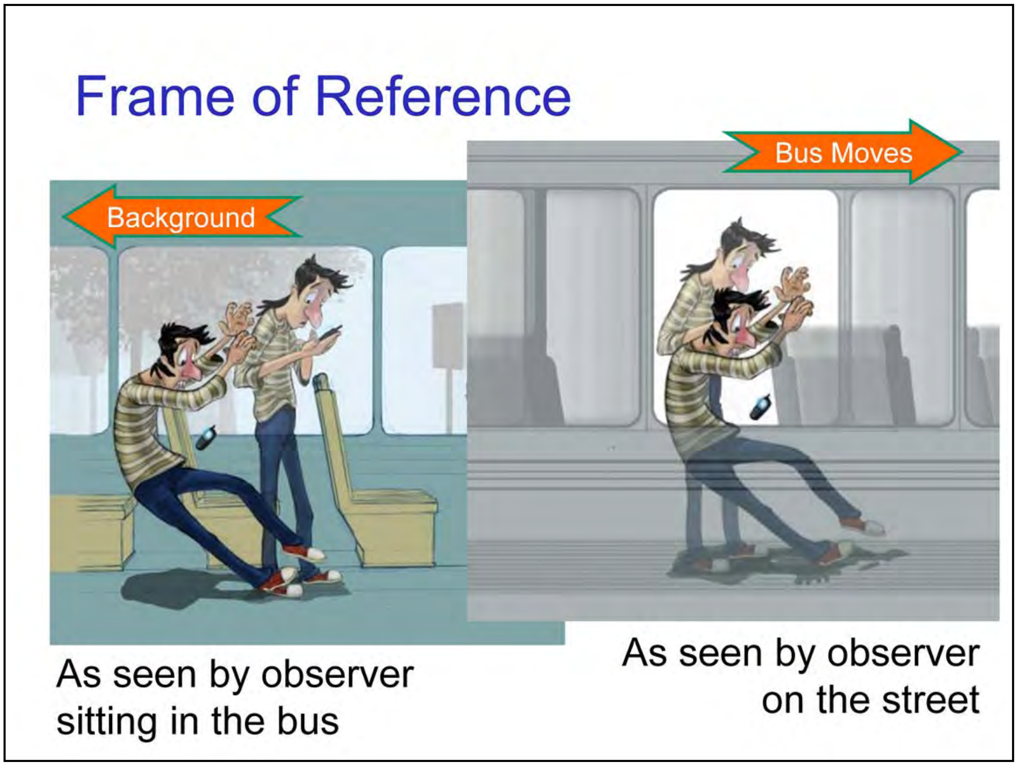
A corollary of the Law of Inertia is that a character at rest (not moving) will remain at rest until acted on by an unbalanced force. Being “at rest” is a special case of uniform motion with speed equal to zero.

Illustration by Charlene Fleming



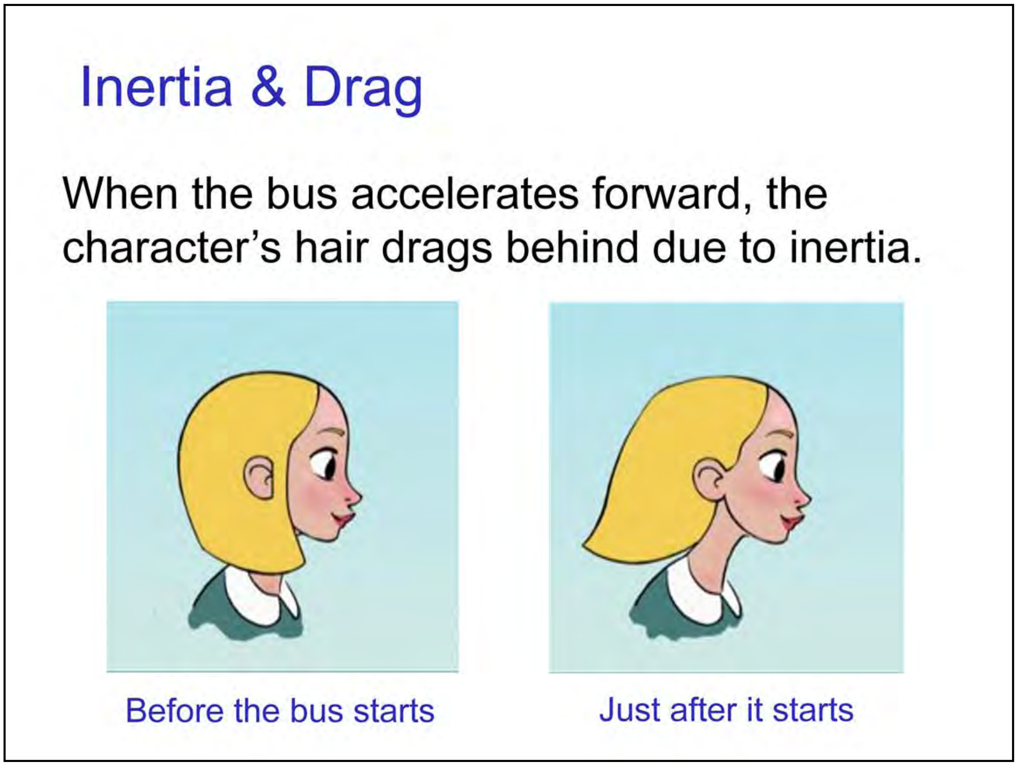
If the bus suddenly accelerates forward then our standing character falls on his back, seemingly thrown backwards towards the rear of the bus.

Illustration by Rebbaz Royee



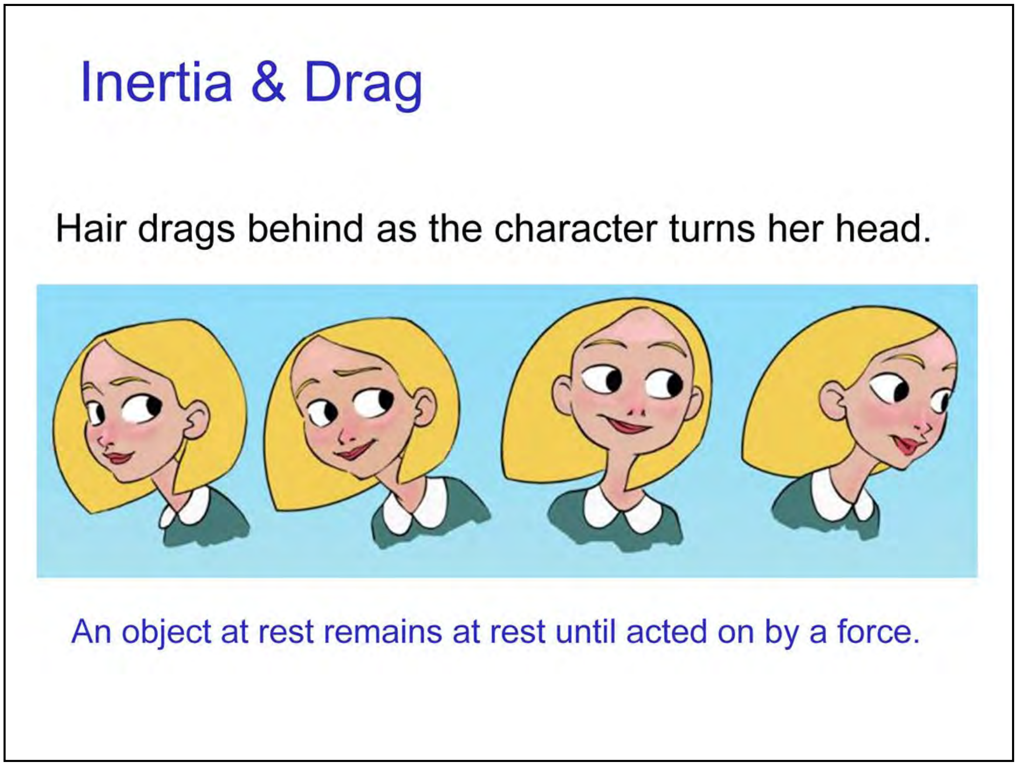
As fellow passengers it seems to us as if there's a force pulling everything backwards but that's because we're moving with the bus. A stationary observer standing outside the bus would realize that the poor chap that's falling has the bus moving out from under him. This reminds us that you have to be careful to include the effect of the camera's motion, especially when the camera is accelerating.

Illustrations by Rebbaz Royee



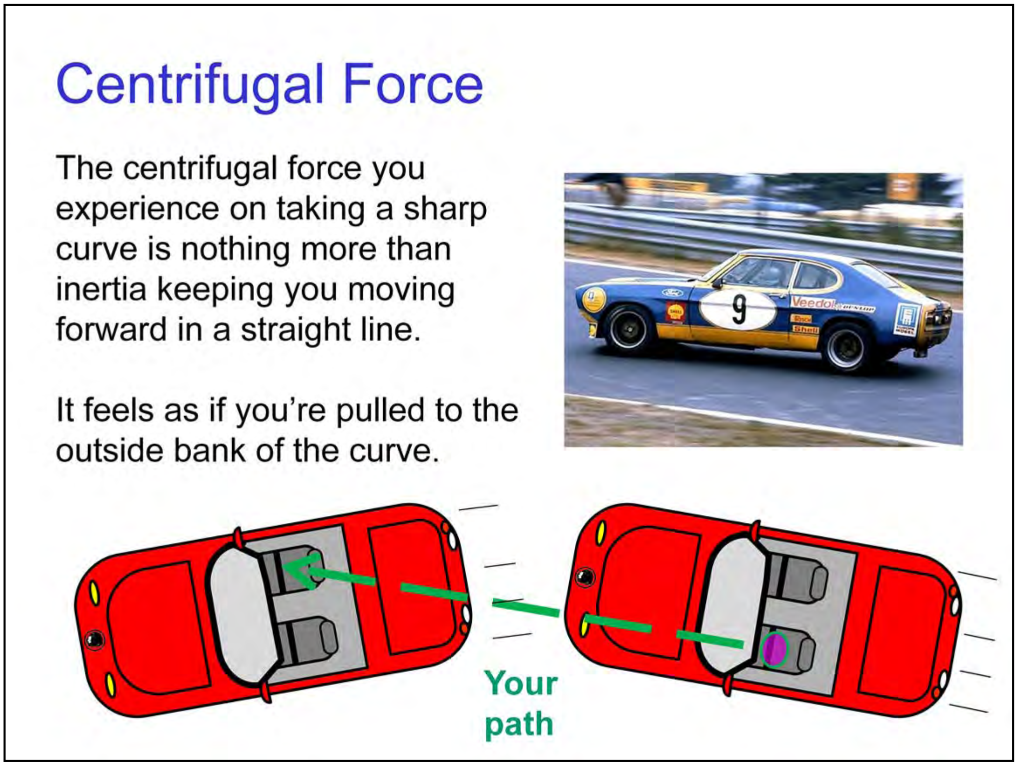
When the bus accelerates forward, the character’s hair drags behind due to inertia.

Illustration by Charlene Fleming



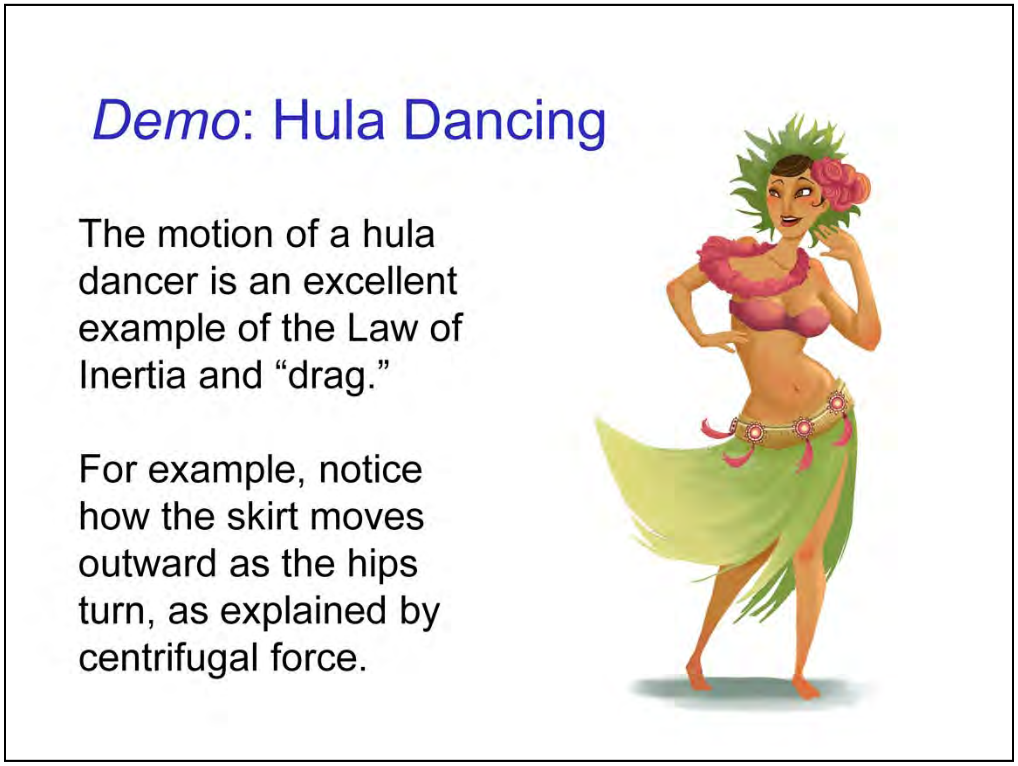
Finally, the Law of Inertia also explains the drag and outward ``centrifugal'' force that's illustrated in Figure~\ref{HairTurnFig}. As the characters turn their bodies their hair and clothing drag behind due to the Law of Inertia (an object at rest will remain at rest until acted on by an unbalanced force). Once the hair and clothing start moving they'll continue moving, causing them to fly outward due to inertia (or, if you prefer, due to follow-through).

Illustration by Charlene Fleming



The centrifugal force you experience on taking a sharp curve is nothing more than inertia keeping you moving forward in a straight line. Although it feels as if you're pulled to the outside bank of the curve it's really the car that's moving toward the inside bank coming towards you. In the same fashion, if the bus makes a sudden right turn then the character standing on the bus will continue moving in his original direction, causing him to fall towards the left side of the bus.

Sports car photo: http://commons.wikimedia.org/wiki/File:Heyer,_Hans_-_Ford_Capri_-_08.07.1973.jpg



The motion of a hula dancer is an excellent example of the Law of Inertia and animation "drag." For example, if the dancer simply swings her hips from side to side then the skirt simply drags behind due to inertia. However if she rotates her hips around in a circle then the skirt moves outward, as explained by centrifugal force.

Illustration by Dora Roychoudhury

Part I: Principles of Animation Physics

#3: Momentum and Force

Momentum

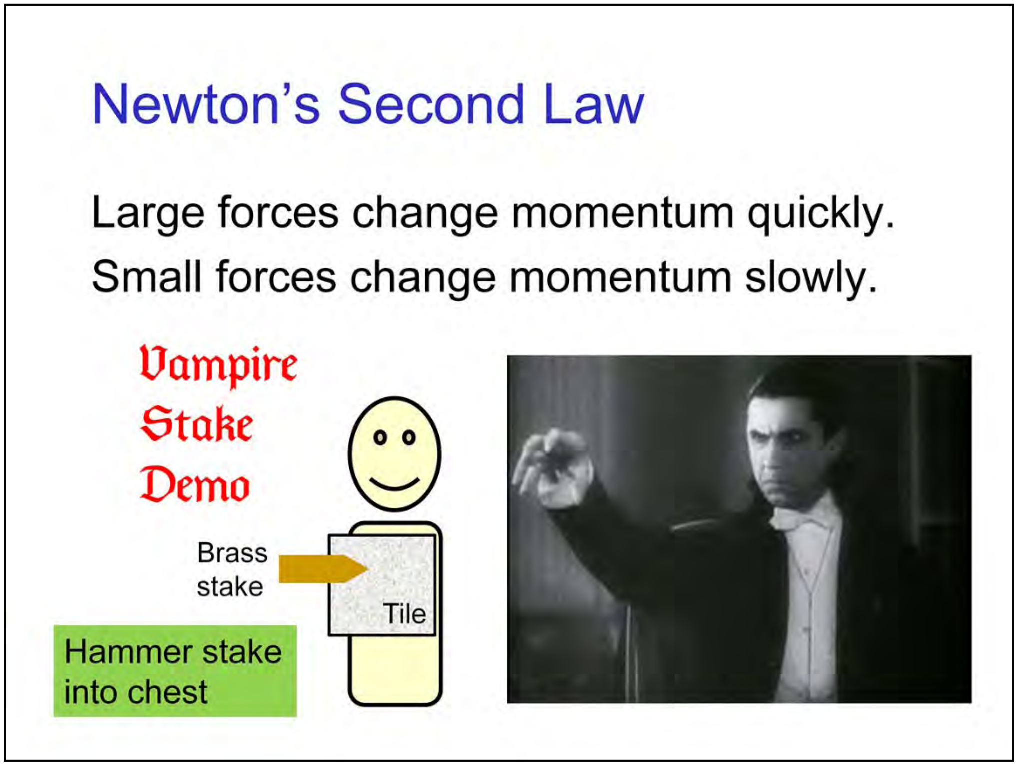
Momentum of an object depends on:

* Velocity of the object
* Weight of the object

A moving character has momentum, which depends on the character's speed and on its weight. A character that weighs 100 pounds can have as much momentum as a 300 pound character if the small character runs three times faster than the big guy. A bullet has a large momentum due to it’s high speed; a rugby player runs a hundred times slower but weighs over a 1000 times more than a bullet so his momentum is even larger.

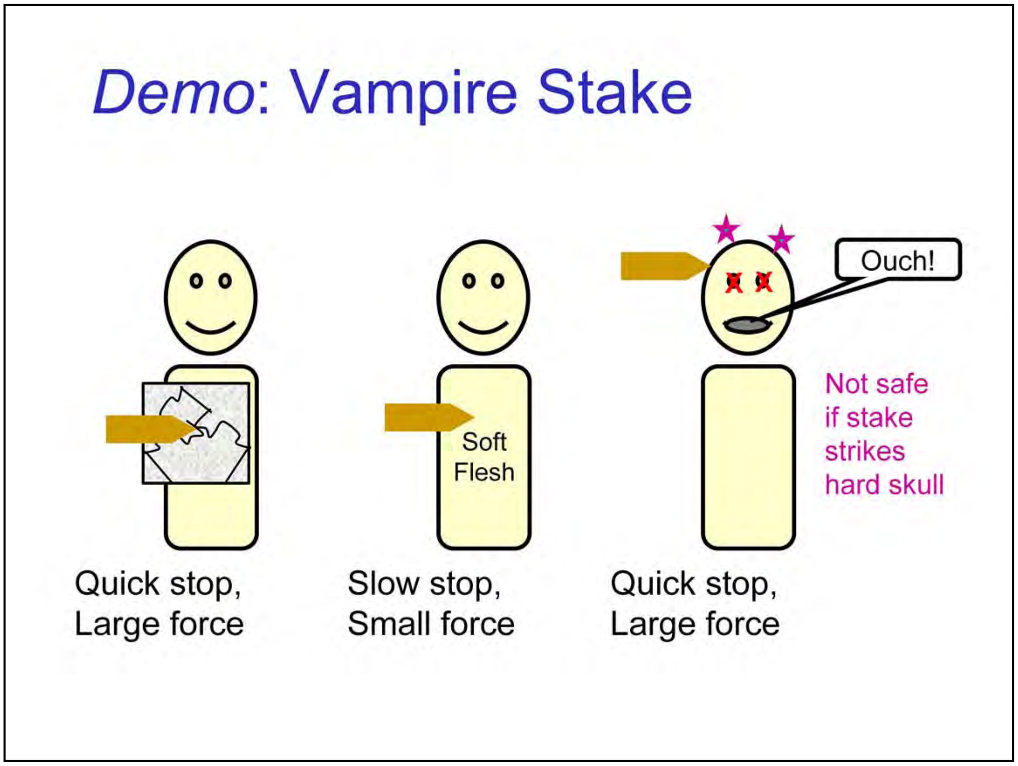
Gun photo: http://commons.wikimedia.org/wiki/File:Bullet_coming_from_S%26W.jpg

Rugby player photo: http://commons.wikimedia.org/wiki/File:Alipate_Tuilevuka_Churchill_Cup_2010_vs_Russia.jpg

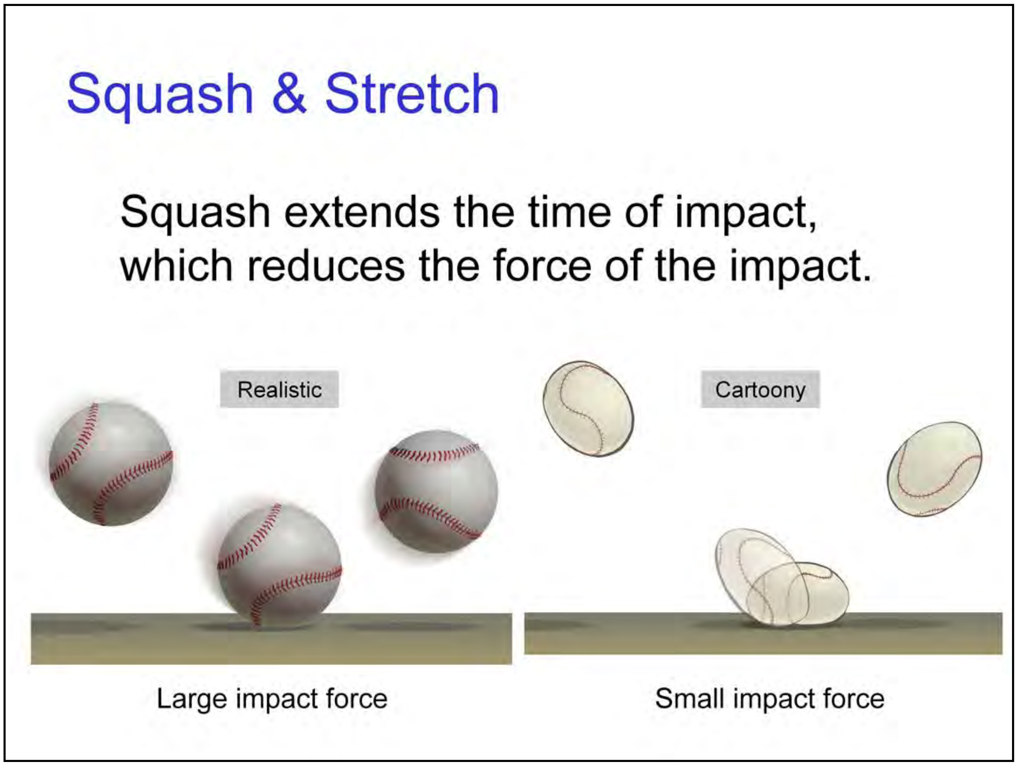


Large forces change momentum quickly while small forces change momentum slowly. This is how Newton originally presented his Second Law of Motion. A dramatic demonstration of this principle is the vampire stake demo in which a brass stake is hammered into my chest.

Dracula photo: http://commons.wikimedia.org/wiki/File:Bela_Lugosi_as_Dracula-2.jpg



When the massive brass stake is hammered into the ceramic tile the stake's momentum changes very quickly since it comes to an immediate stop; the force is very large, enough to crack and break the tile. On the other hand, when the stake is hammered into my soft chest the momentum change is much slower and thus the force required to stop it is much less. This demo is only safe when the stake is placed against soft flesh and with a massive stake so that it's velocity is small (but with enough momentum to crack the tile).

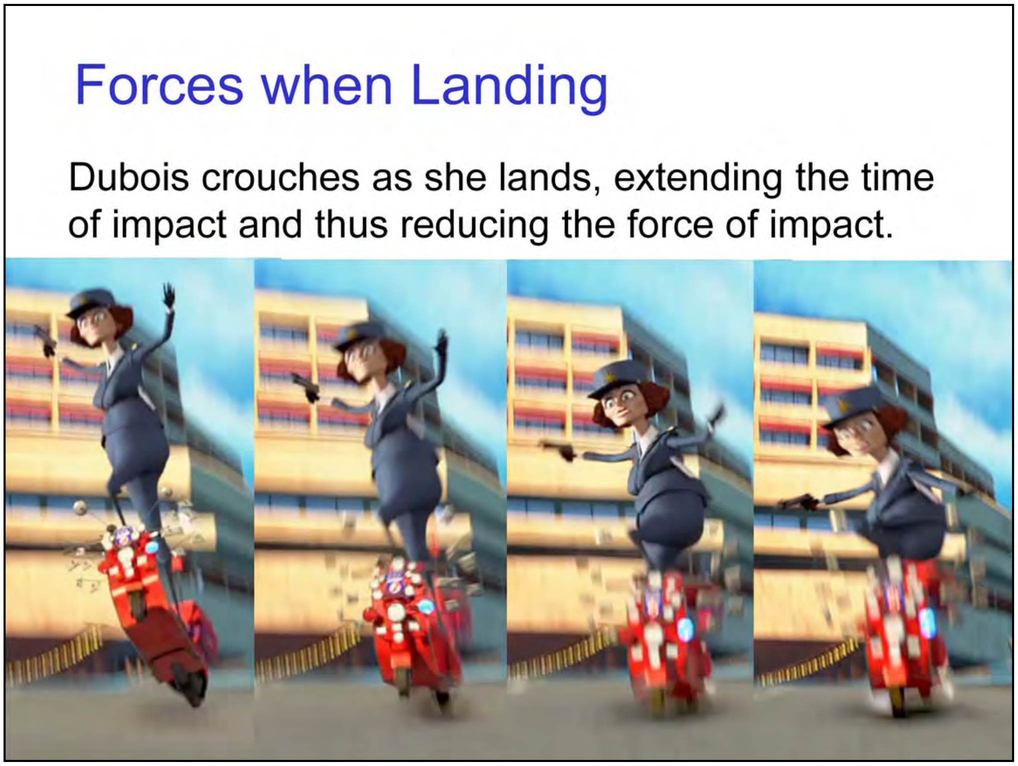


The connection between timing and forces is seen all the time, such as in these two examples of a ball hitting the ground. The momentum change could be about the same in the two cases but in the realistic impact that change occurs quickly, implying a large force at impact. Using ``squash and stretch" the cartoony version softens the impact by having the change in momentum spread over a few frames. In brief, crisp timing means large forces and vice versa.

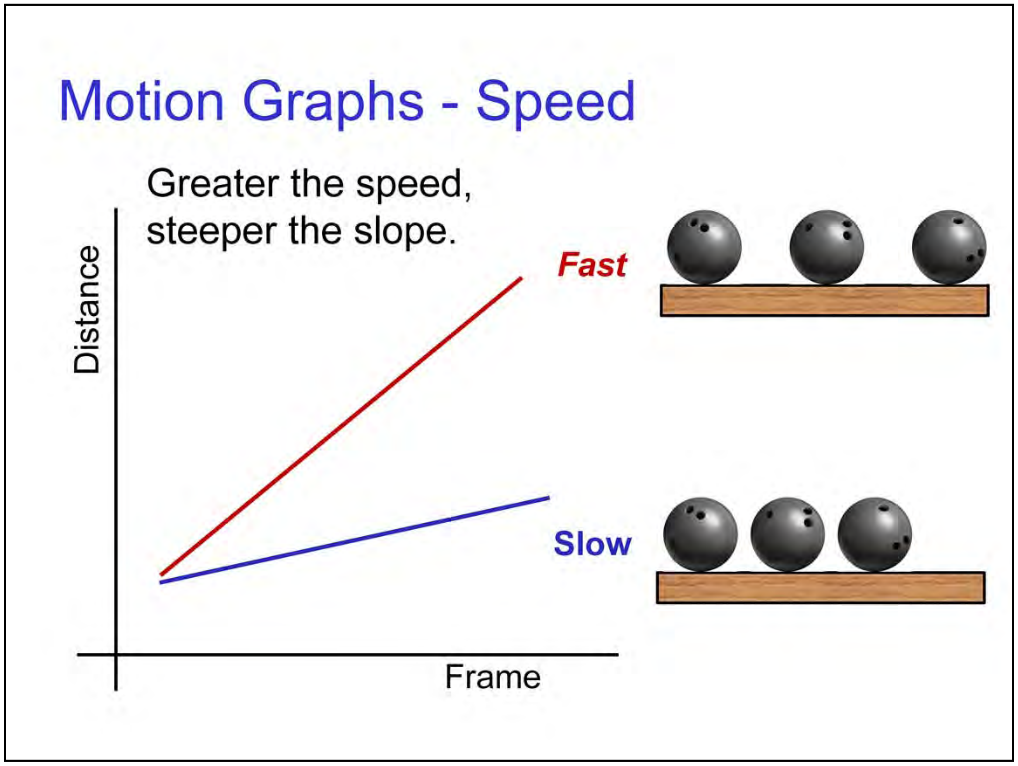
Illustrations by Charlene Fleming and Rebbaz Royee

Forces when Landing

.

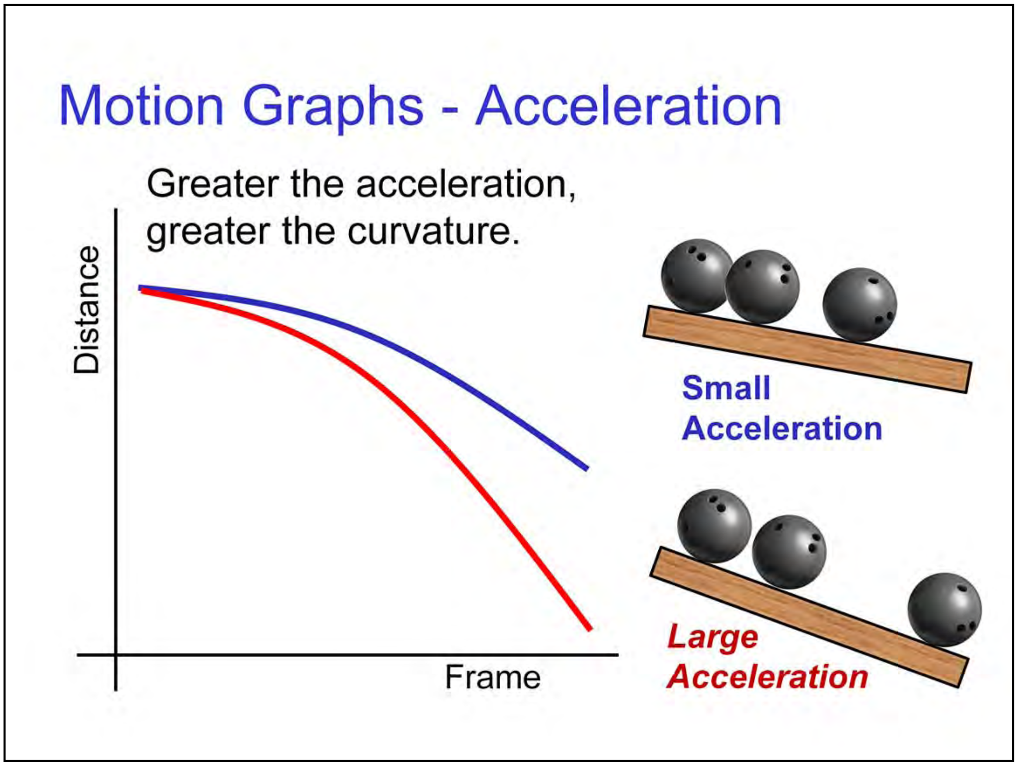


Dubois crouches as she lands, extending the time of impact and thus reducing the force of impact.



The motion graph for an object moving with constant speed is a straight line. The steeper the slope, the greater the speed (and the greater the momentum).

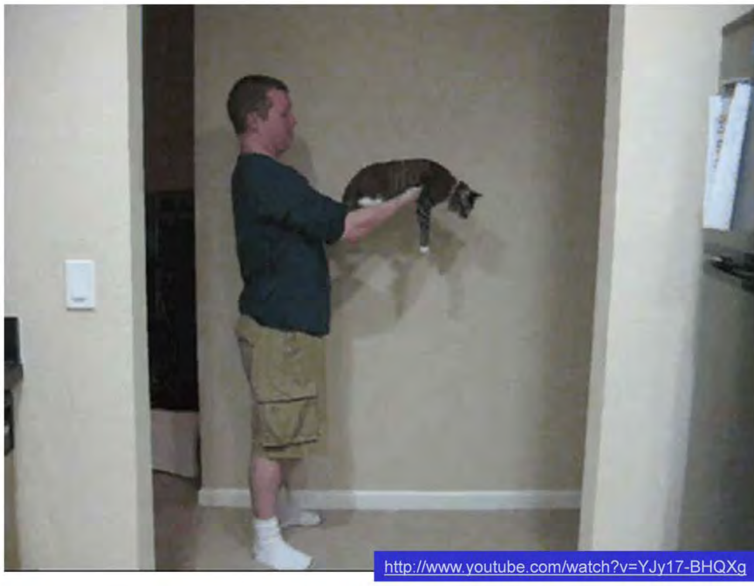
Illustration by Charlene Fleming



For an accelerating object the motion graph is a curved arc. When momentum is changing quickly (i.e., large unbalanced force) the acceleration is large and the curvature is large. If the unbalanced force is constant then the curve is a parabolic arc (see Fourth Down at Half Time).

Illustration by Charlene Fleming

Cat Drop Video Reference



http://www.youtube.com/watch?v=YJy17-BHQXg

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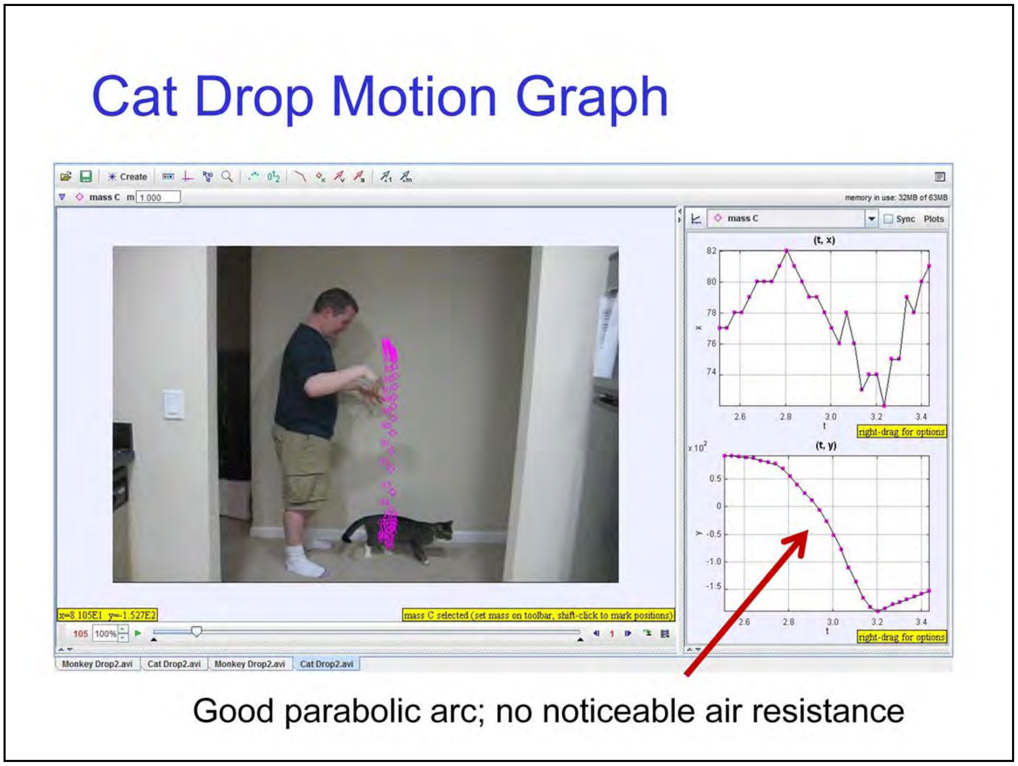
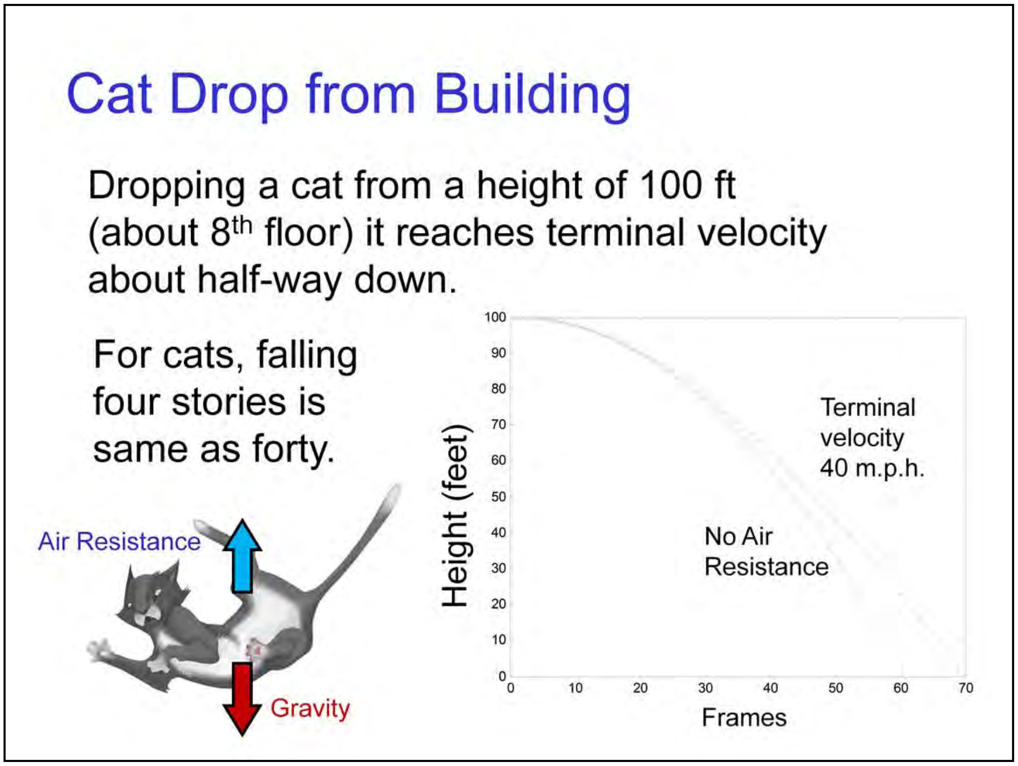
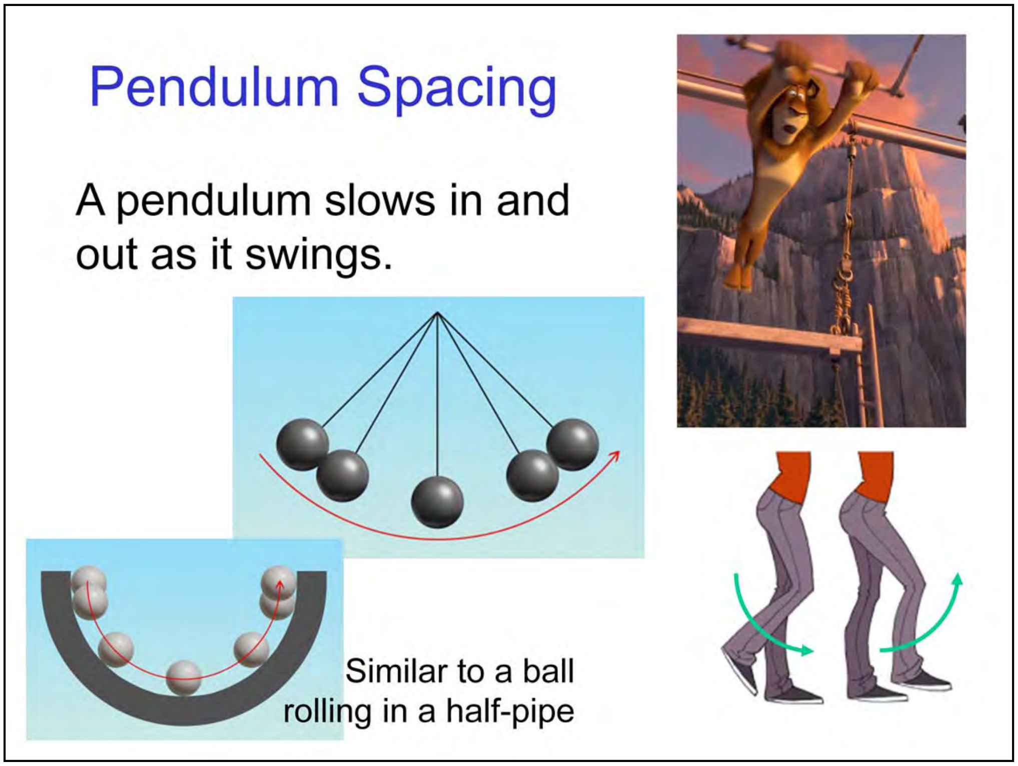


As a class assignment my students shoot video reference of falling objects and analyze the motion using *Tracker*, a public-domain video analysis program. From the motion graph of the vertical position versus time we see that, within experimental error, the curve is a parabolic arc (see Fourth Down at Half Time).



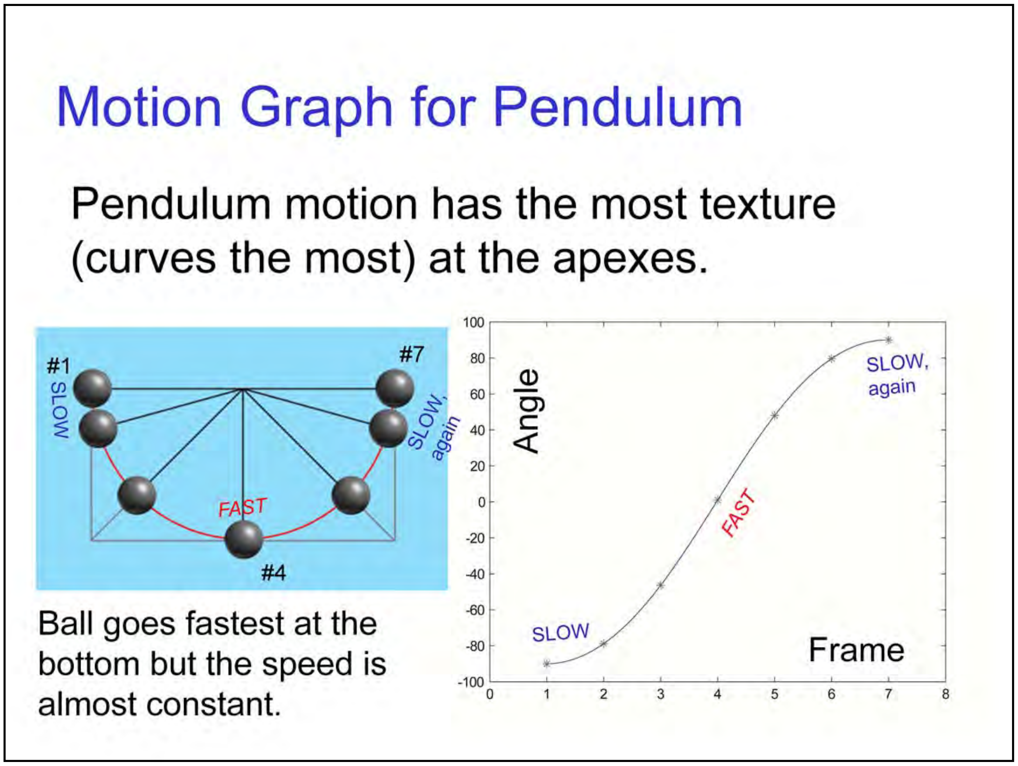
For falls from greater heights the effect of air resistance starts to become noticeable. A falling character will reach a maximum speed (called the terminal velocity) when the force of air resistance balances the force of gravity. At that point the momentum stops changing and the character falls with constant speed. For small creatures, such as squirrels, this terminal velocity is relatively slow so they usually survive falls, regardless of the height.

Illustration by Dora Roychoudhury



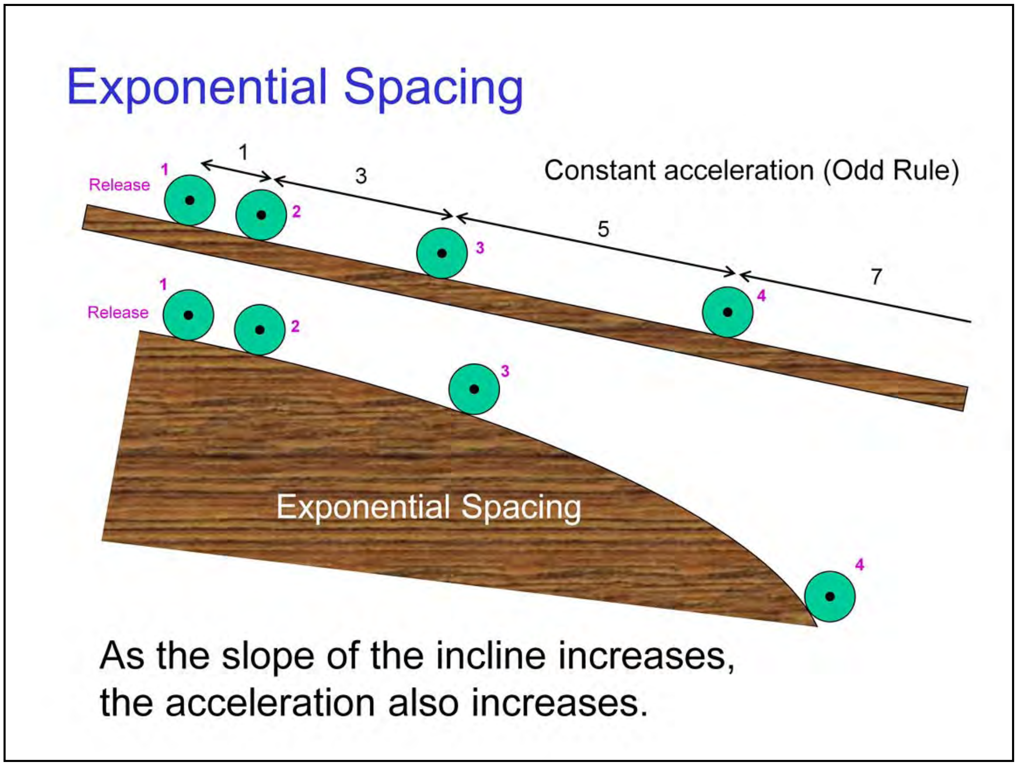
A common example in which the spacings do not exactly follow the “Odd Rule” is the swinging of a pendulum.

Illustrations by Charlene Fleming

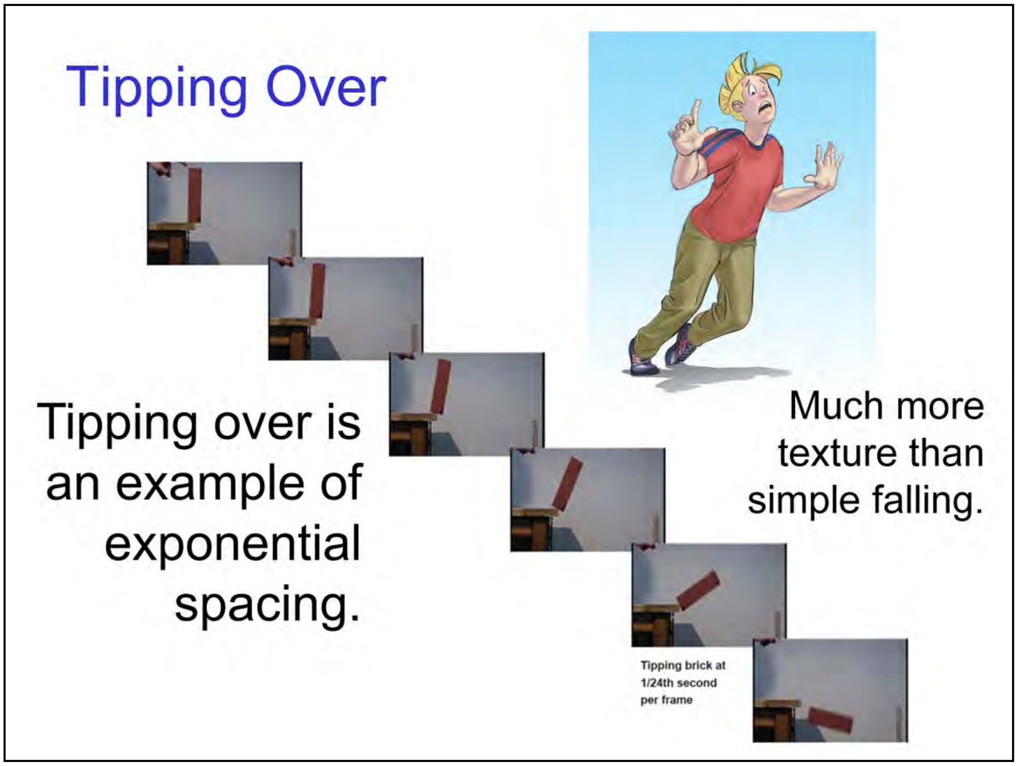


A pendulum slows out of its apex, has its maximum speed at the bottom of its swing, then slows again as it rises to the apex on the opposite side. Although this sounds a bit like the motion of a bouncing ball, the forces are not constant for the swinging pendulum so the timing and spacings have more texture than with the "Odd Rule."

Illustration by Charlene Fleming



Another example for which the spacing differs from the "Odd Rule" occurs when a ball rolls down a curved slope. In this case the unbalanced force accelerating the ball is not constant and the spacings increase faster than a ball rolling down a straight ramp. We'll call this exponential spacing (although the precise form of the spacings varies depending on how quickly or slowly the unbalanced force is changing).



Tipping over is a good example of exponential spacing. Notice the texture in the timing as the brick tips over; at first it hardly moves, creating anticipation, but then the spacings increase rapidly.

Illustration by Rebbaz Royee

Part I: Principles of Animation Physics

#4: Center of Gravity

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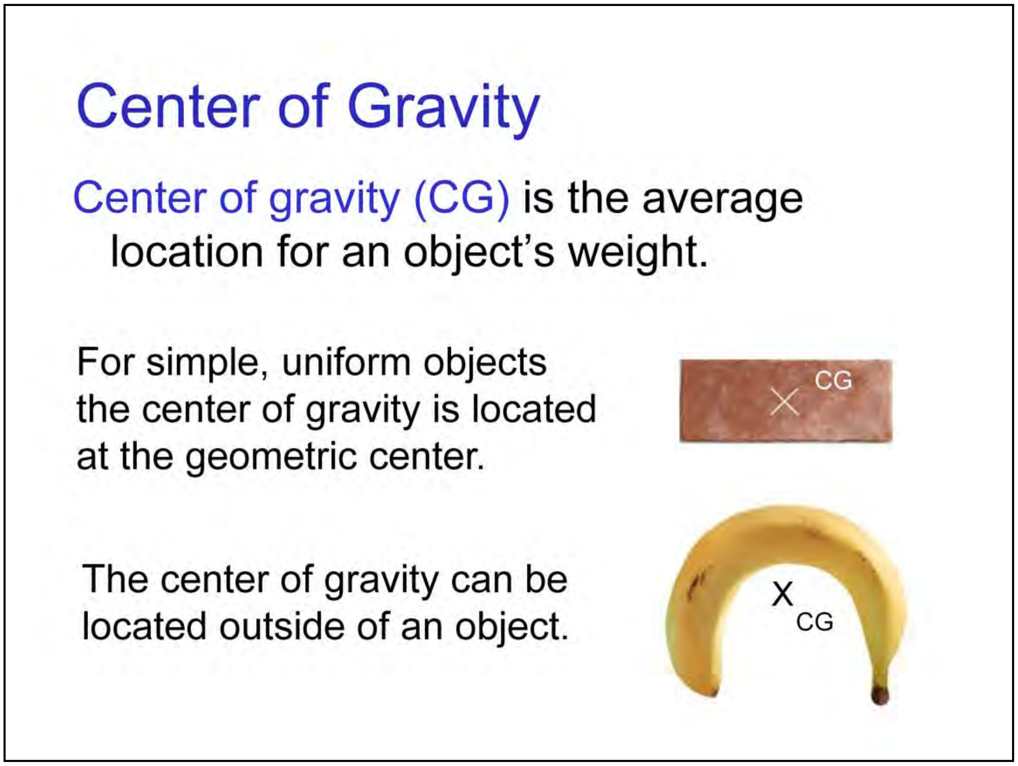
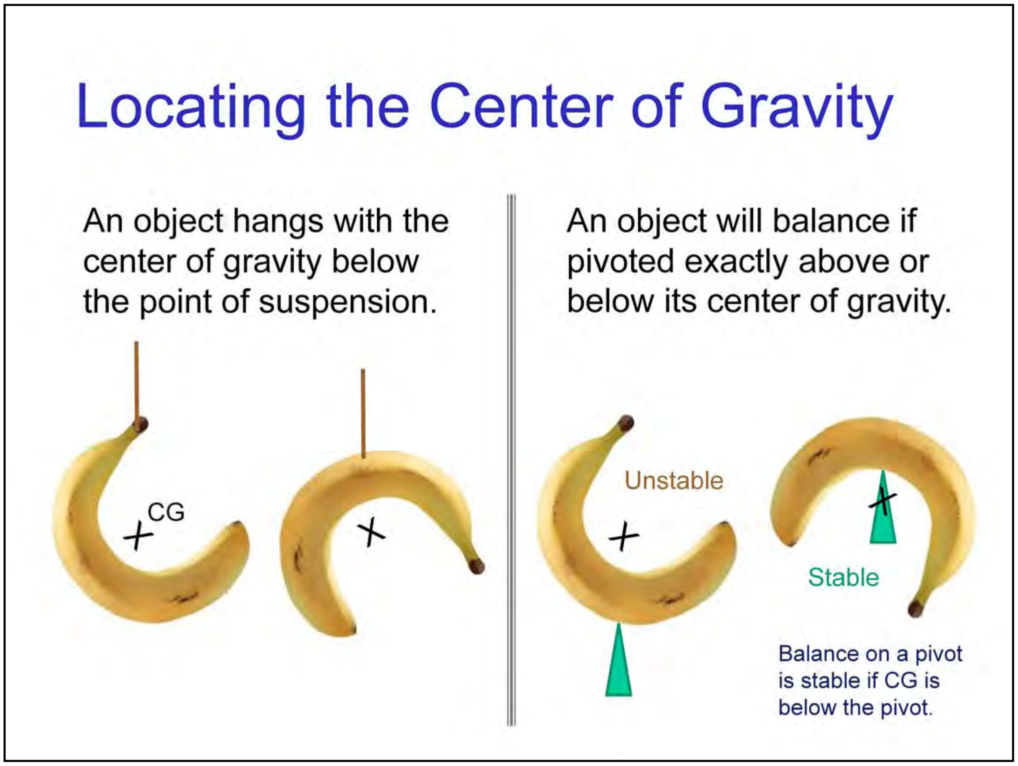


The center of gravity is the average location for an object's weight. For simple, uniform objects the center of gravity is located at the geometric center. This is the geometric (visual) center for most objects but for non-uniform objects (e.g., hammer with a wooden handle and an iron head) it's located closer to the heavier side. The center of gravity can be located outside of an object, for example for a donut the center of gravity is located in the middle of the hole. If an object changes shape, for example if a character changes pose, then the center of gravity will change location.

Illustrations by Charlene Fleming



The center of gravity can be computed using calculus but it can also be determined by experiment. For example, an object hangs with the center of gravity below the point of suspension; by using two hanging configurations we can locate the center of gravity by triangulation (i.e., the intersection of the two plumb lines). An alternative approach is to balance the object on a sharp pivot; the center of gravity will be directly above (or below) the pivot if the balance is unstable (or stable). Often it's enough simply to imagine these situations to get an estimate for the location of the center of gravity.

Illustration by Charlene Fleming

Center of Gravity and Hanging

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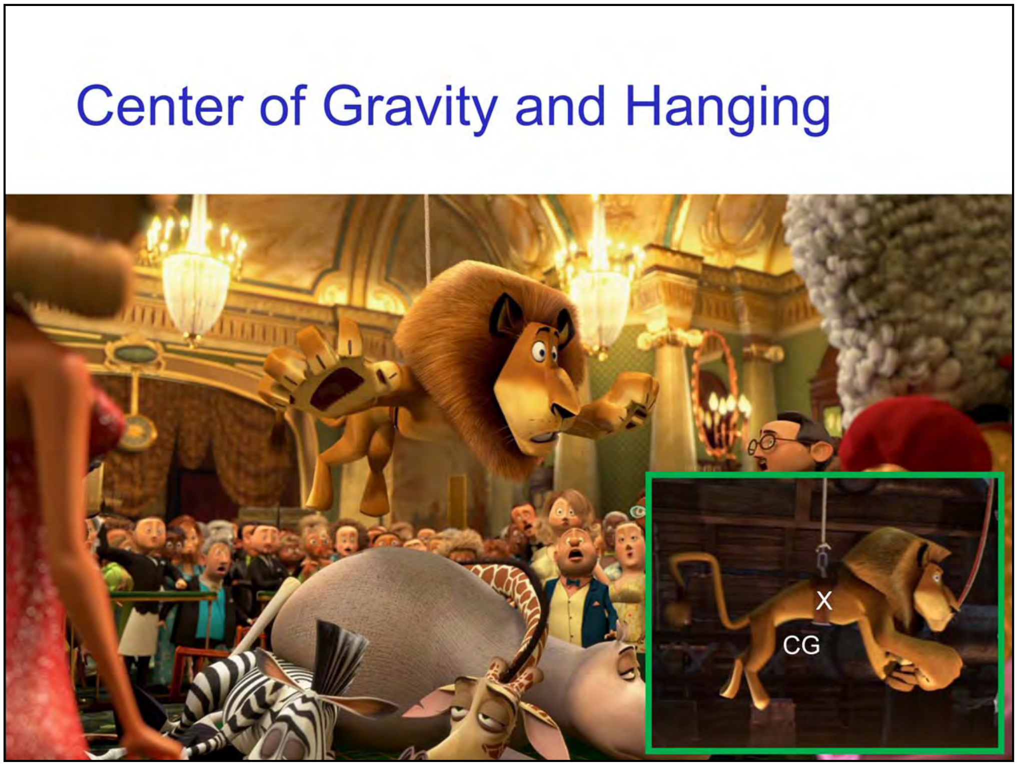
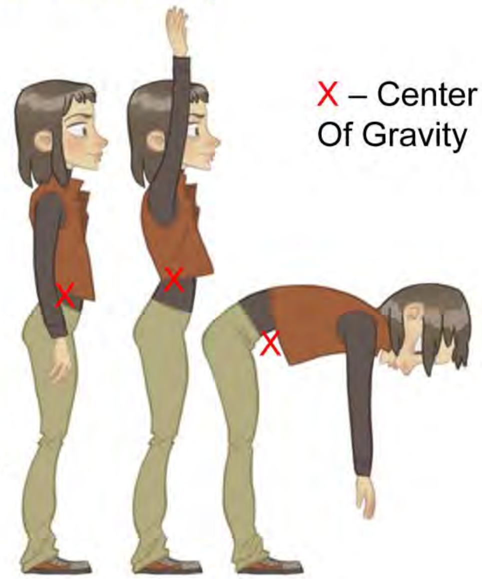


The harness by which Alex the Lion is hanging needs to be around his chest so that his center of gravity is under the point of suspension while in this horizontal pose. In a similar scene from the movie Mission Impossible (1996) the harness used by Tom Cruise is around his hips but Alex the Lion is top-heavy so his center of gravity is positioned up in his chest.

Human Center of Gravity

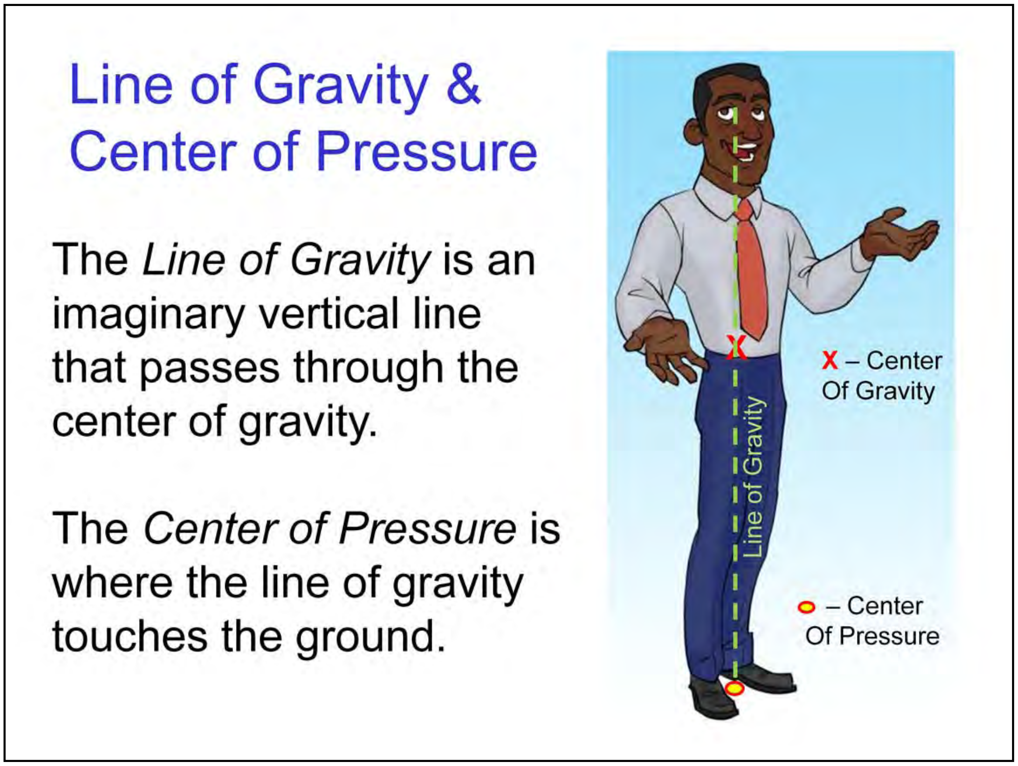
Standing upright, your CG is roughly in the center of your body (at about 55% of your height).

Location of your CG will shift when you bend your torso, move your arms and legs, etc.



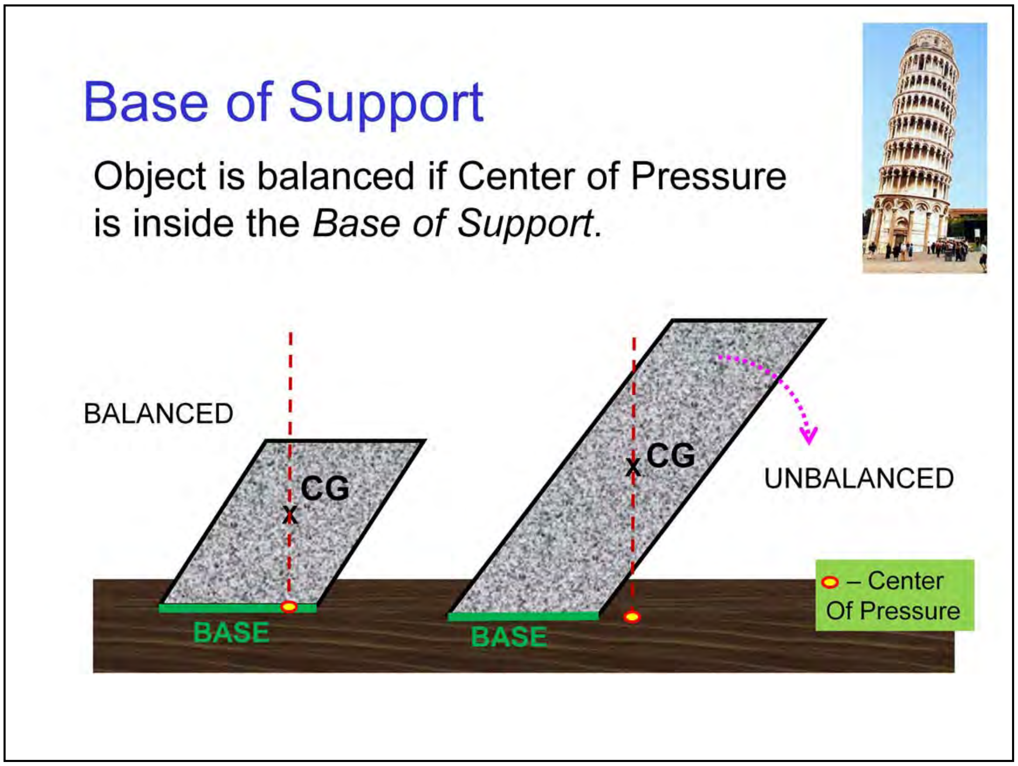
The center of gravity is important for various reasons. First, a character is in a balanced stationary pose only if the center of gravity is positioned over the character's feet. More precisely, the center of gravity has to be over the “base of support”, which is the area around the feet. For a person standing upright the center of gravity is roughly in the center of the body (actually just slightly above the center due to the weight of the head). The location of the center of gravity can shift, for example if you raise your arms it shifts upward. Bending forward the center of gravity can actually be positioned outside of your body.

Illustration by Charlene Fleming



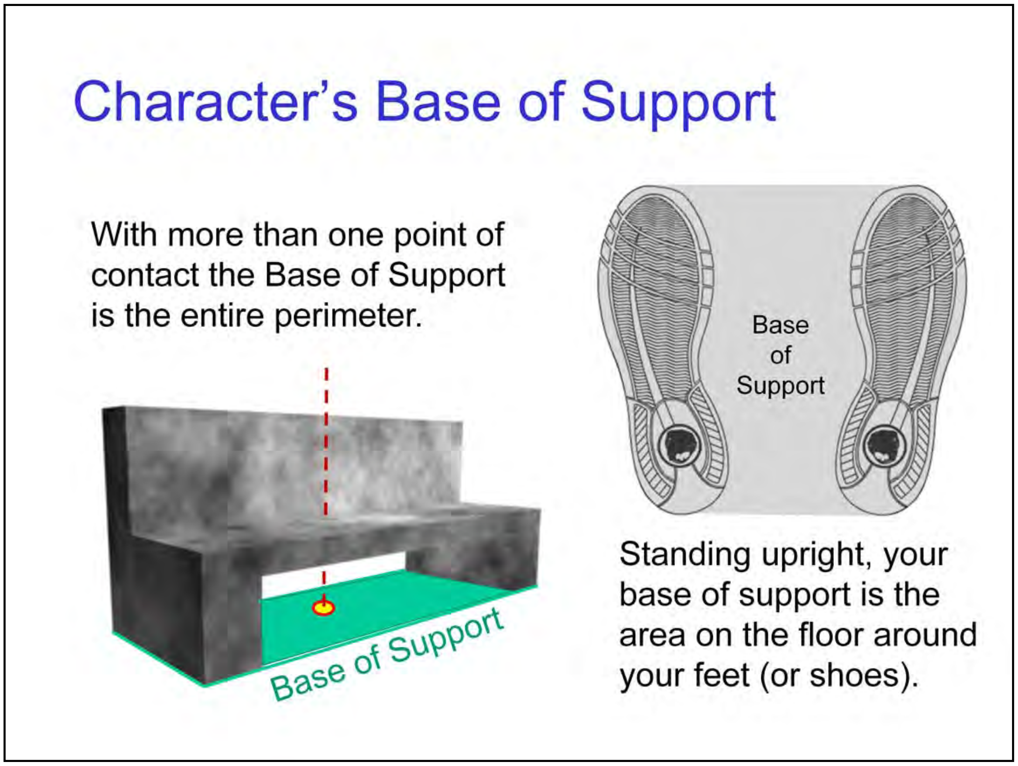
When working with the center of gravity it's useful to use the line of gravity, which is an imaginary vertical plumb line that passes through the center of gravity. It's also useful to consider the center of pressure, which is the place where the line of gravity touches the ground.

Illustration by Rebbaz Royee



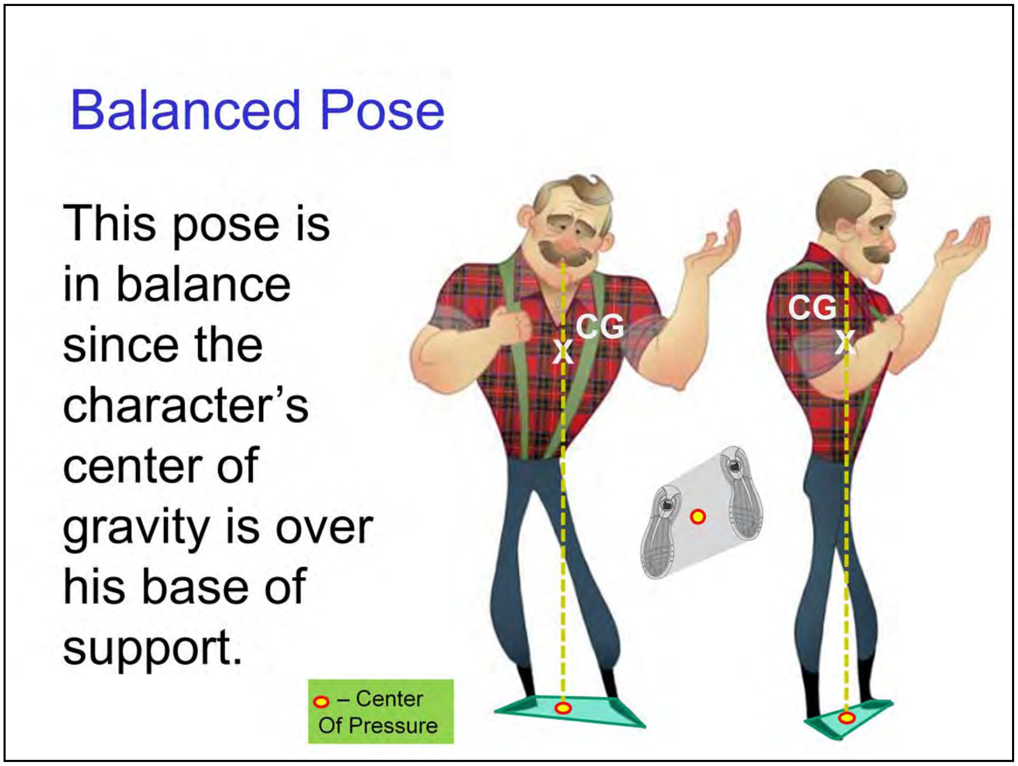
An object is in balance when the center of pressure is inside the base of support. For simple objects, like a tilted cylinder, the base of support is just the area of contact with the floor.

Tower photo: http://commons.wikimedia.org/wiki/File:Leaning_tower_of_pisa_4.jpg



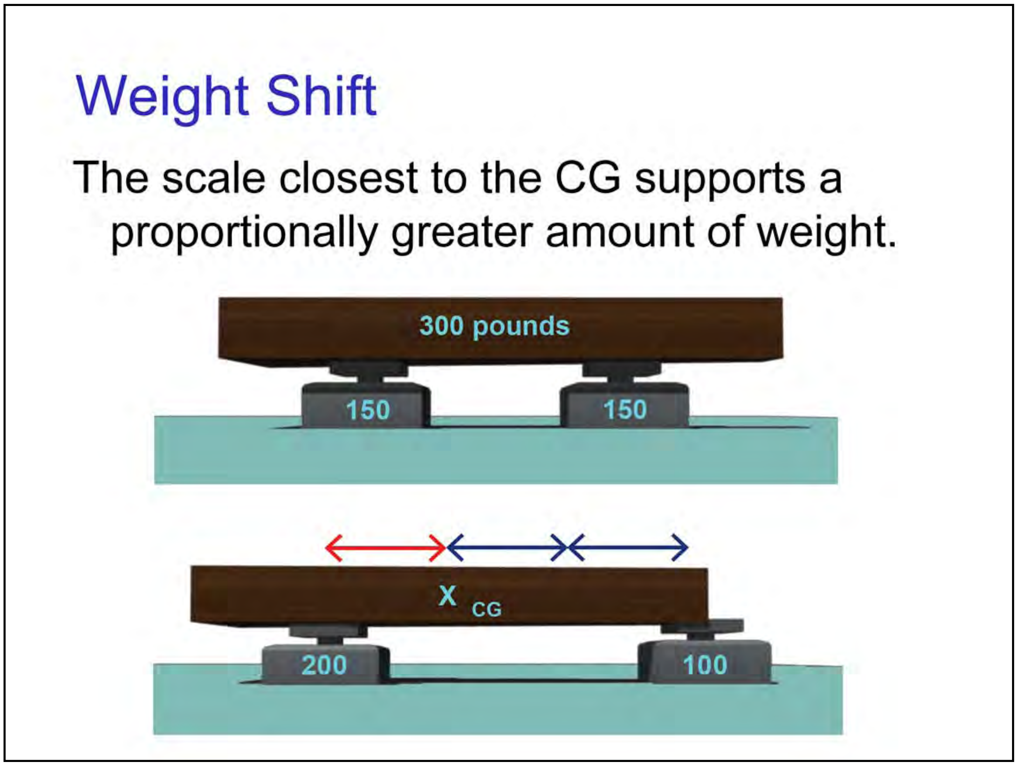
With more than one point of contact, such as a bench with two legs, the base of support is the entire perimeter that encircles all the points of contact. Standing upright your base of support is the area on the floor around your feet (or shoes). Quadrupeds have a large base of support since it is the entire area around their feet.

Illustrations by Corey Tom



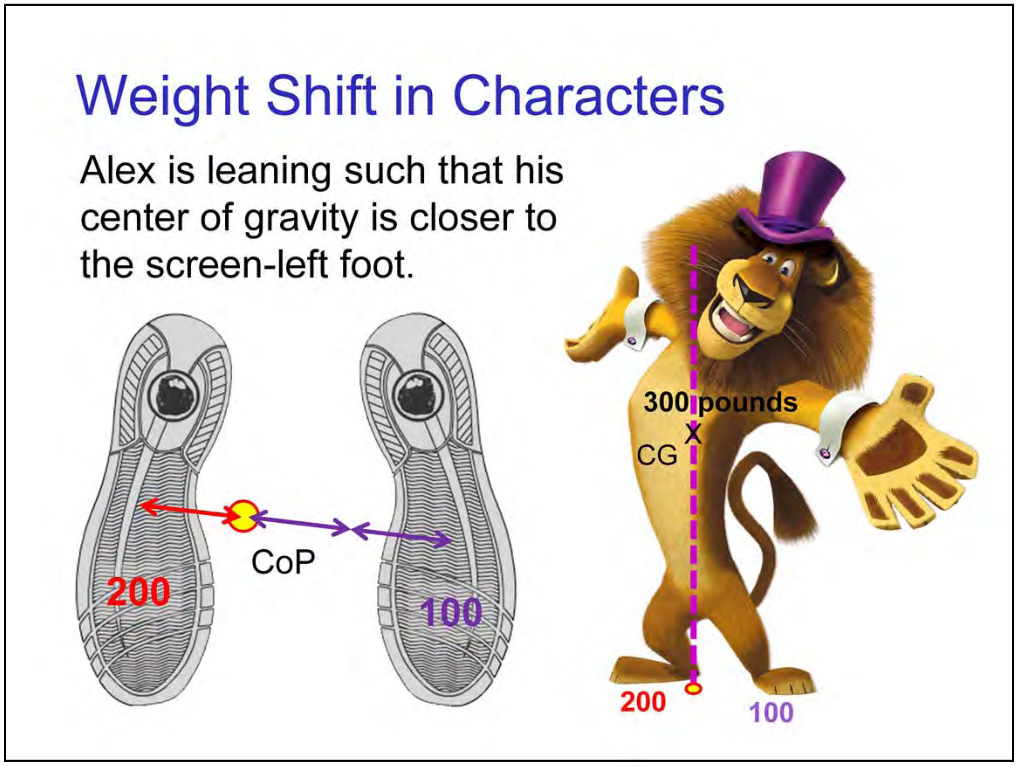
A character is in a balanced stationary pose only if the center of gravity is positioned over the character's feet. More precisely, the center of gravity has to be over the "base of support"', which is the area around the feet. Note that this character does not have a normal human physique so the center of gravity is positioned higher.

Illustration by Charlene Fleming



When an object is supported over several points of contact the weight is distributed according to the position of the center of gravity. For example, when the 300 pound slab is centered between two supports then each carries 150 pounds of weight. But if we shift the center of gravity so that it's twice as far from one support than from the other then the weight shifts. The support closer to the center of gravity now carries twice the weight of the other, that is, it carries 200 pounds and the other carries 100 pounds.

Illustration by Corey Tom

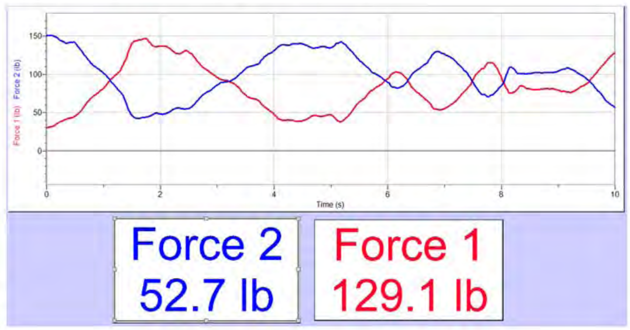


When a character changes pose the character’s center of gravity shifts, causing a weight shift from one leg to another. Simply shifting the center of gravity by a few inches is enough to cause significantly weight shift.

Shoe illustration by Corey Tom

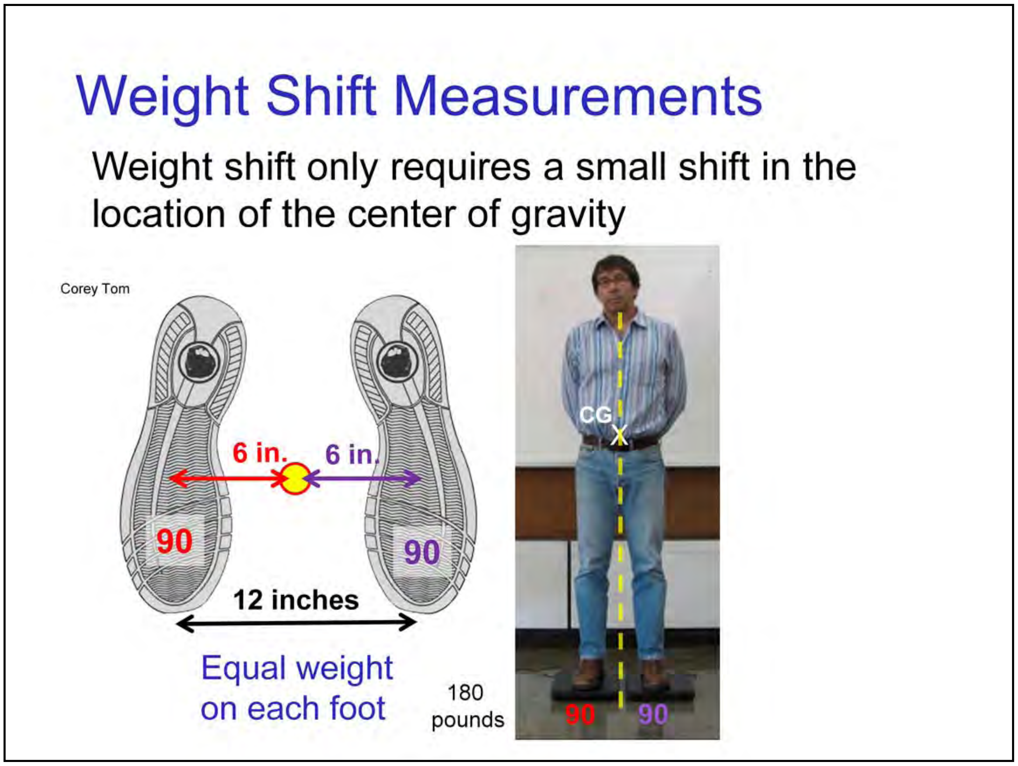
Demo: Weight Shift on Force Plate

Force plates are essentially digital bathroom scales that can be connected to a computer for collecting data.



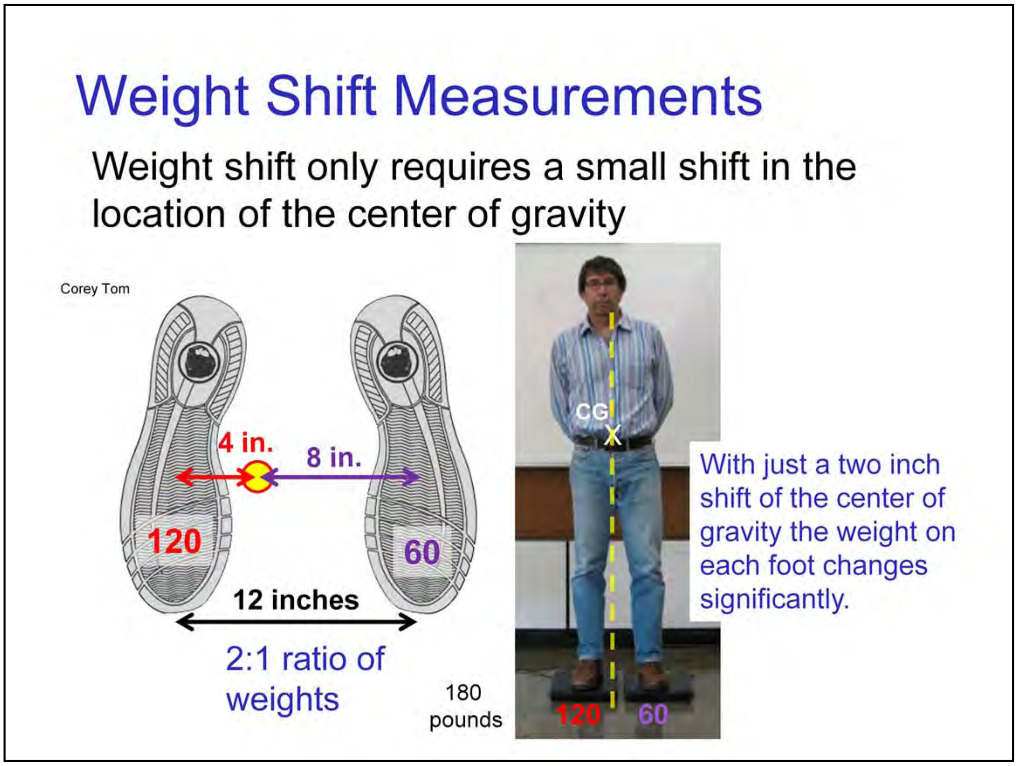
Variation in weight is surprisingly large.

We can see how much weight is on each leg by using force plates. Interestingly, even small changes in a person’s pose create very significant shifts in weight.



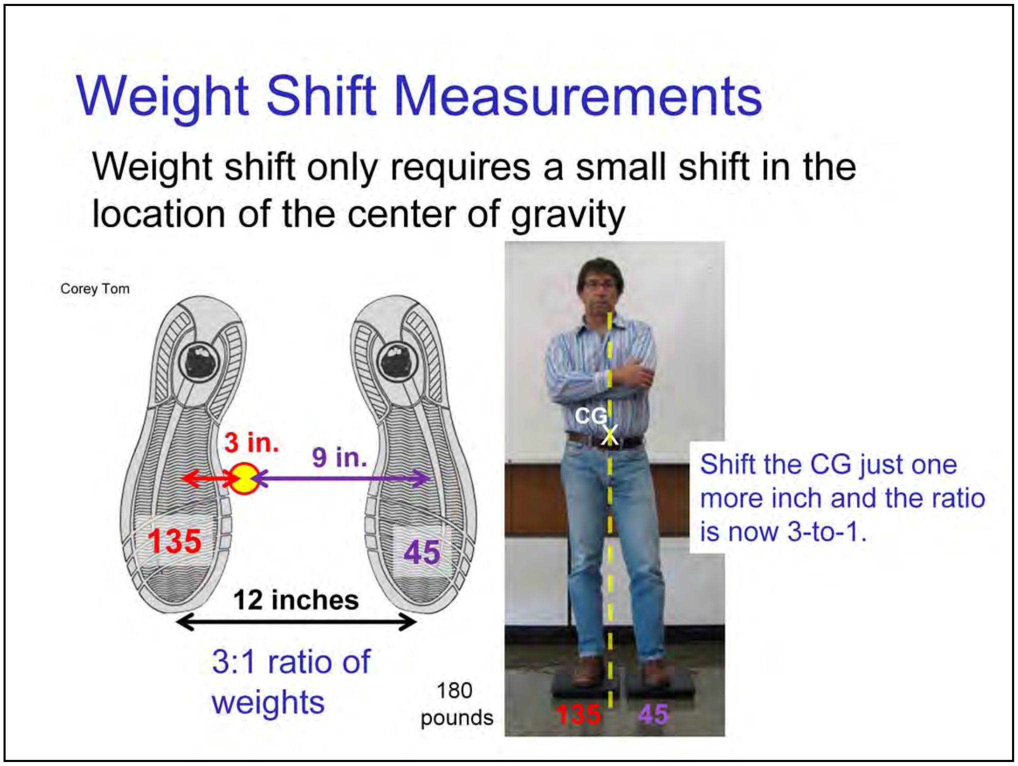
In this pose the weight on each leg is about the same; the center of gravity is positioned an equal distance from each foot.

Illustration by Corey Tom



Shifting the center of gravity by just two inches increases the weight to that side significantly; now the weight on the screen left foot is twice that of the weight on the other foot.

Illustration by Corey Tom



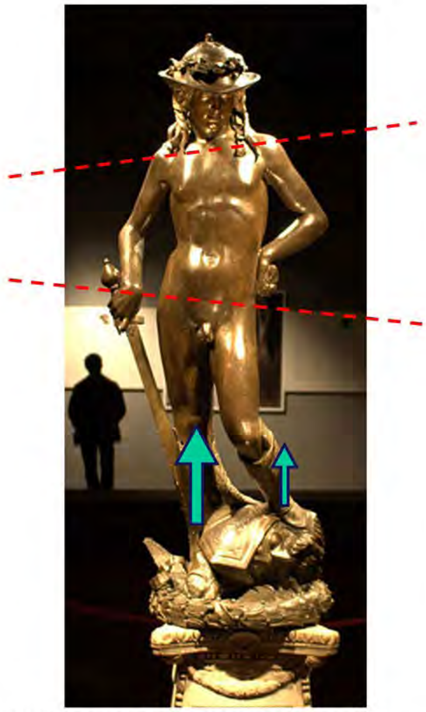
Shifting the center of gravity by one more inch now gives a 3-to-1 ratio of weights; notice the changes in the pose as the weight shift occurs.

Illustration by Corey Tom

Contrapposto

Weight shifts from foot to foot are reflected by corresponding shifts in the hips and shoulders.

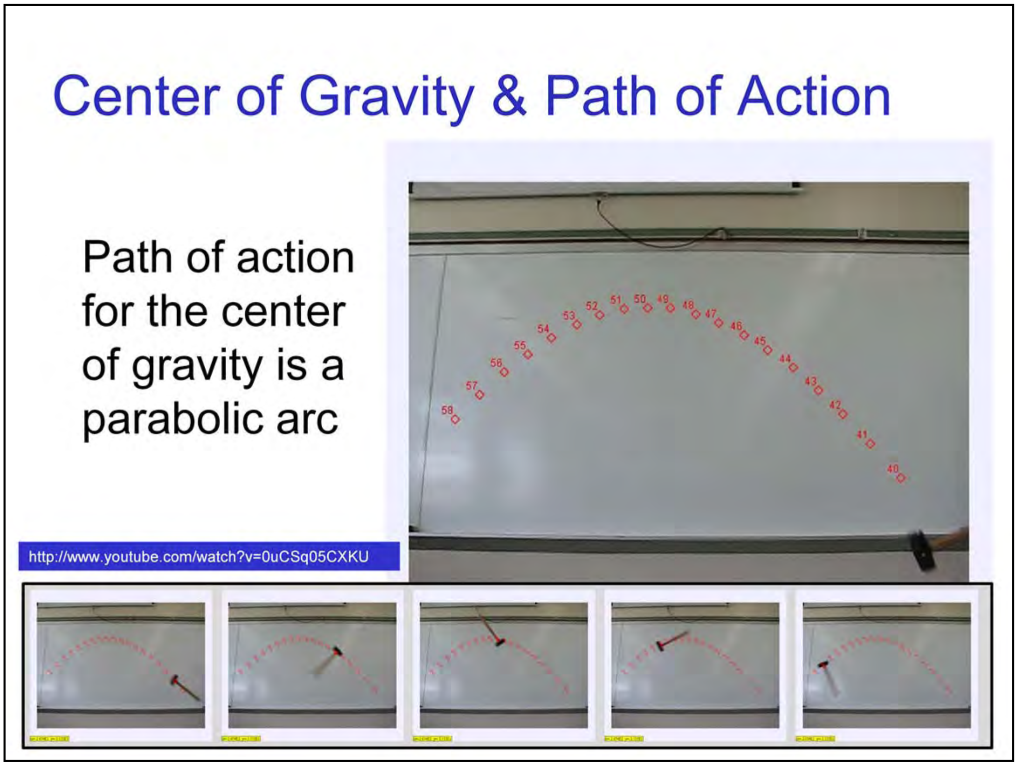
Notice the angles of the hips and shoulders and how they converge on the side that bears more of the weight.



Donatello's bronze statue of David (circa 1440s)

Weight shifts from foot to foot are reflected in the pose, typically raising the hip and lowering the shoulder on the weight-bearing side (an effect known as ``contrapposto"). When animated well it should be clear whether a character is standing or sitting even if the shot only shows the character's upper body.

Photo of Donatello’s David:
http://commons.wikimedia.org/wiki/File:David_di_donatello,_mostra_a_milano_03.2.jpg



When we consider the path of action of a moving character the point that we're most interested in tracking is the center of gravity. For example, when flying through the air it is the center of gravity that follows a parabolic arc, independent of any rotation in the body or twisting of the limbs. The motion of the center of gravity is also important in walks, as we'll discuss later.

Part I: Principles of Animation Physics

#5: Weight Gain and Loss

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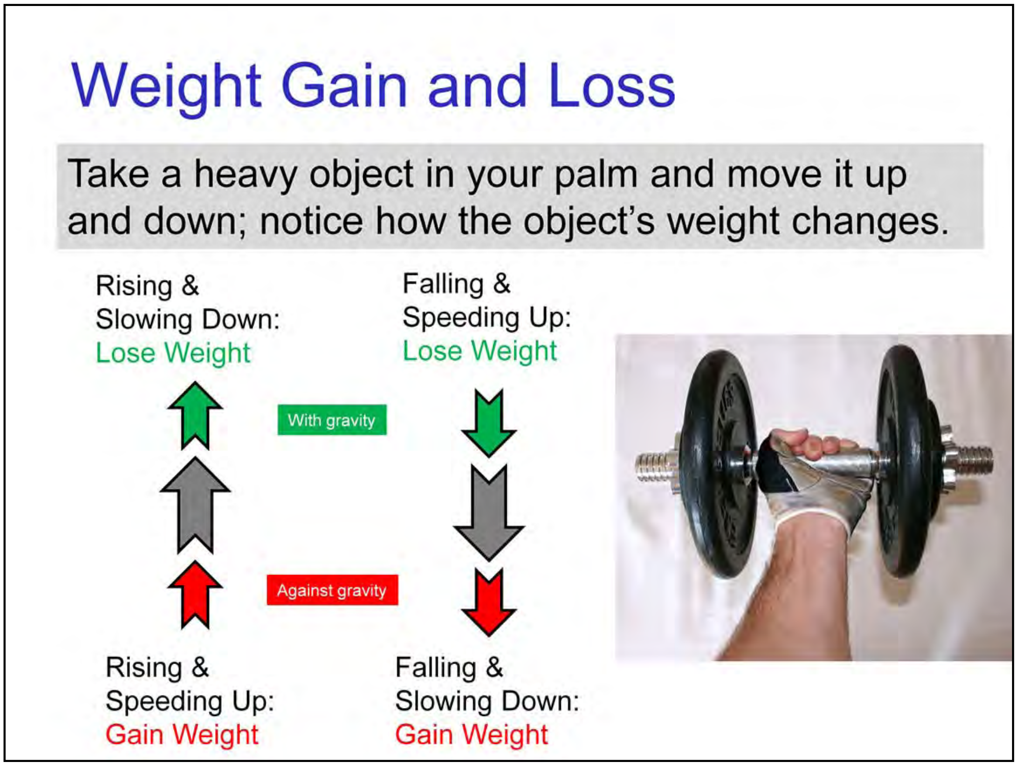
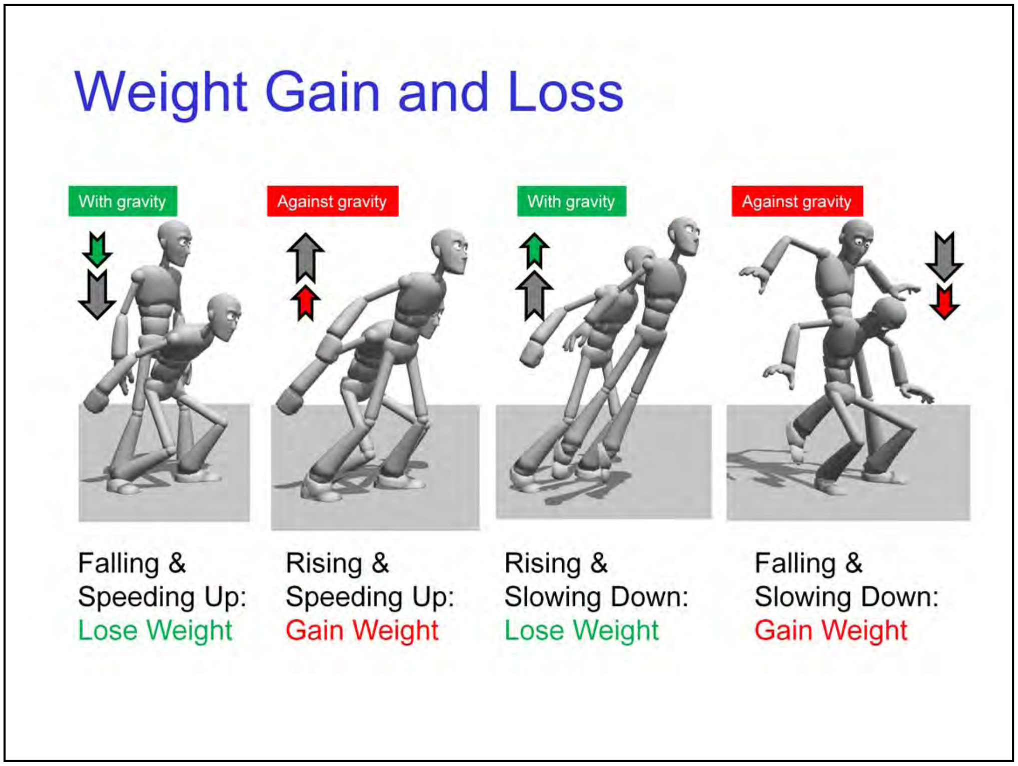


We think of an object's weight as being constant but when the object accelerates its weight effectively varies. Specifically, if the object is:

- Moving upward and gaining speed: Gain weight
- Moving upward and losing speed: Lose weight
- Moving downward and gaining speed: Lose weight
- Moving downward and losing speed: Gain weight

This weight gain and loss is easily demonstrated if you take a heavy object, say a dumbbell or a large water bottle, and move it up and down. The object should be heavy enough so that you notice it's weight, not just feel the contact with your skin. Another example is lifting a heavy bag of groceries by the bag's handle; if you quickly pull upward the weight gain can be large enough to break the handle.

Dumbbell photo: http://commons.wikimedia.org/wiki/File:Kurzhantel_untergriff_15kg0.jpg



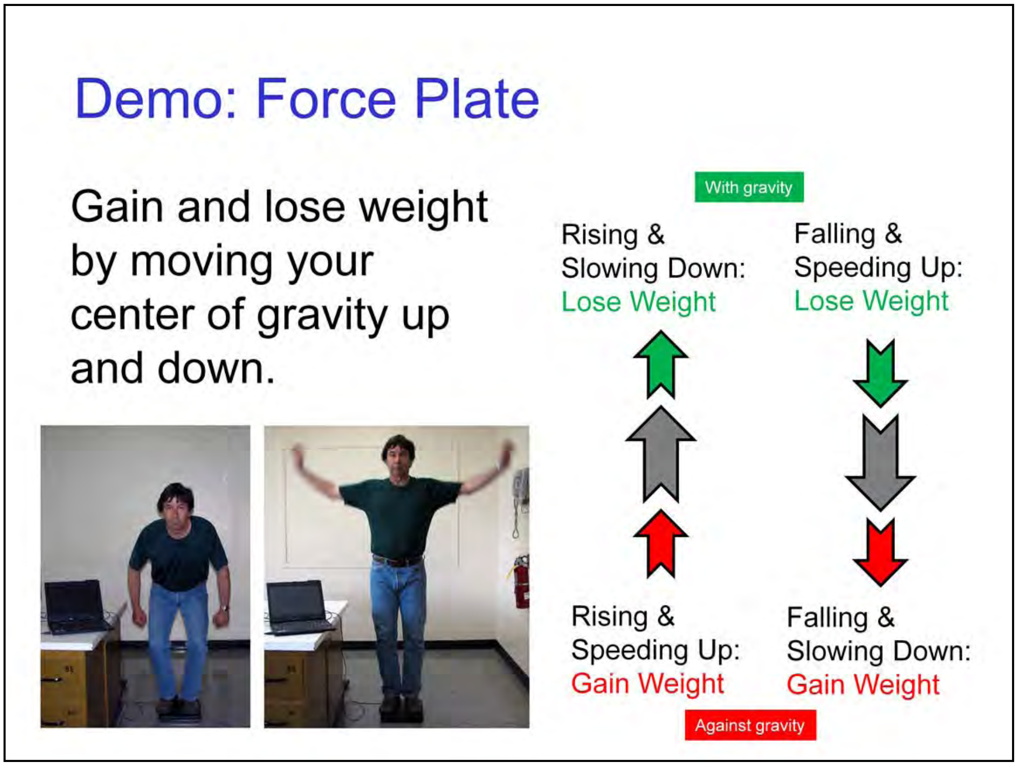
A character's effective weight varies during the different phases of a jump. As the character starts dropping into the crouch the weigh drops; as the character pushes upward gaining speed the weight increases. When the character leaves the ground and is in the air the weight drops down to zero. Finally, on landing the character gains weight as he decelerate; the quicker he come to a stop, the greater the weight gain as he lands.

Illustration by Corey Tom

NASA has a special airplane with a padded interior for training astronauts in a weightless environment. During free-fall the weight goes to zero and people float as if there was no gravity. The plane is called the "Vomit Comet" because after a few seconds of freefall the plane pulls up sharply out of its dive, climbs, repeats the process and this extreme roller-coaster motion can be sickening.

Airplane photo: http://commons.wikimedia.org/wiki/File:Vomit_Comet.jpg

Astronauts photo:
http://commons.wikimedia.org/wiki/File:Astronauts_in_weightlessness.jpg



The effect of gaining and losing weight during motion can be demonstrated using force plates.

“Counteraction”

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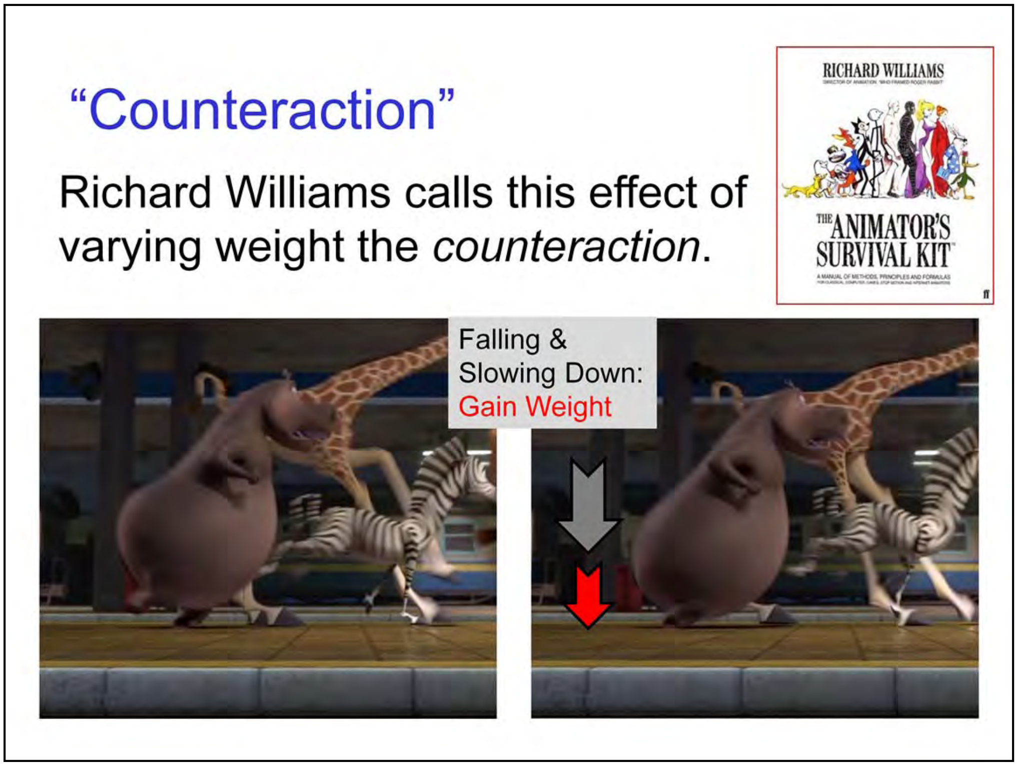
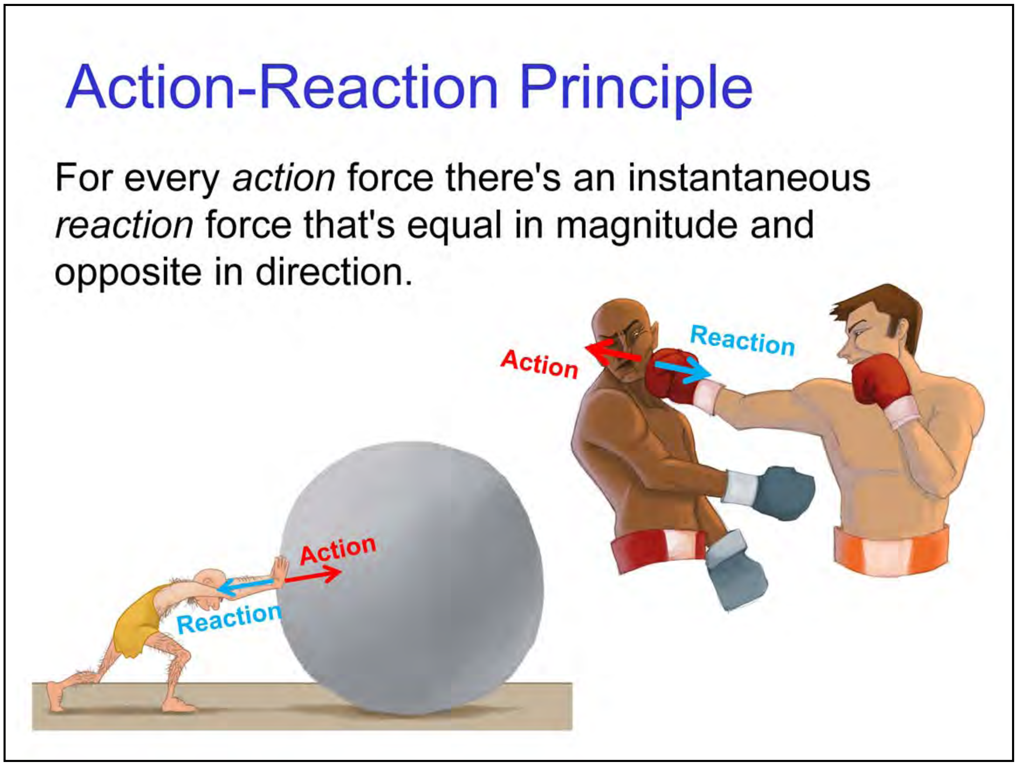


As a character moves (walks, runs, jumps, etc.) the changes in weight create many overlapping actions in the movement of hair, clothing, and flesh. Gaining weight pulls these downward while losing weight causes them to almost float. Richard Williams calls this effect “counteraction” and in *The Animator's Survival Kit* he writes, “When the character (accelerates) up – the drapery or hair or soft bits go down.” Poorly animated characters sometimes look “floaty” as they walk because their actions lack this variation in weight.

Book cover: http://en.wikipedia.org/wiki/File:Animsurvivalkit.jpg

Part I: Principles of Animation Physics

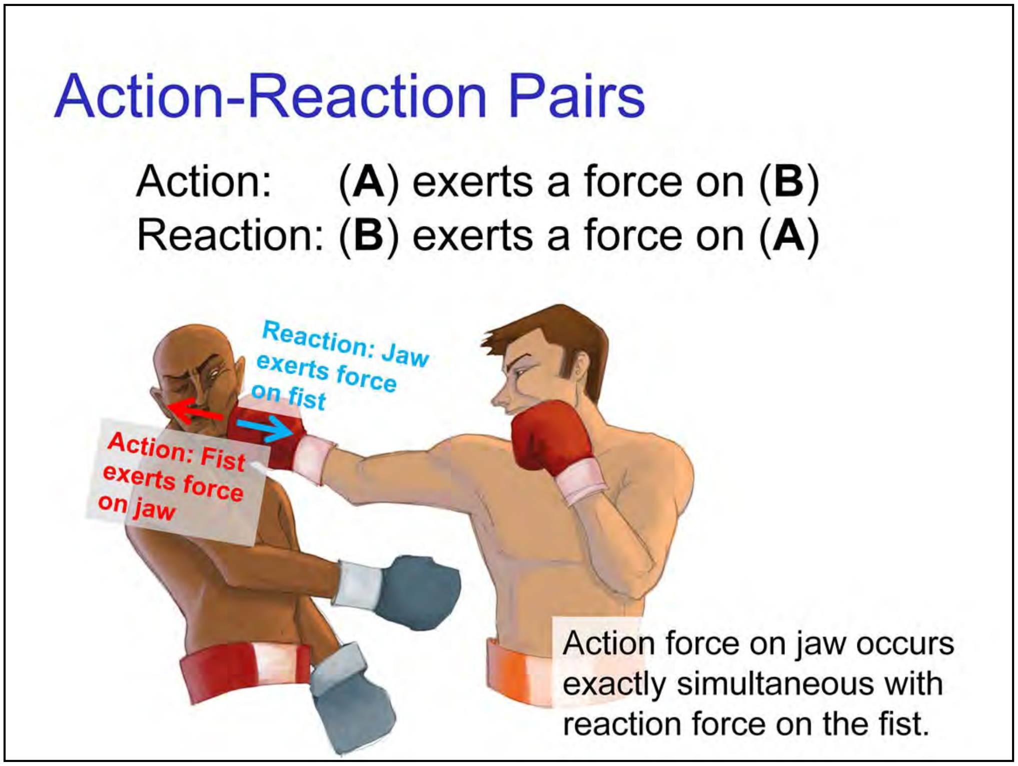
#6: Action and Reaction



The principle of action-reaction, also known as Newton's Third Law, tells us that when a character interacts with an object (or with another character) both are affected. Paul Hewitt poetically describes this as, “You cannot touch without being touched.” A more traditional statement of the principle would be,

“For every *action* force there's an instantaneous *reaction* force that's equal in magnitude and opposite in direction.”

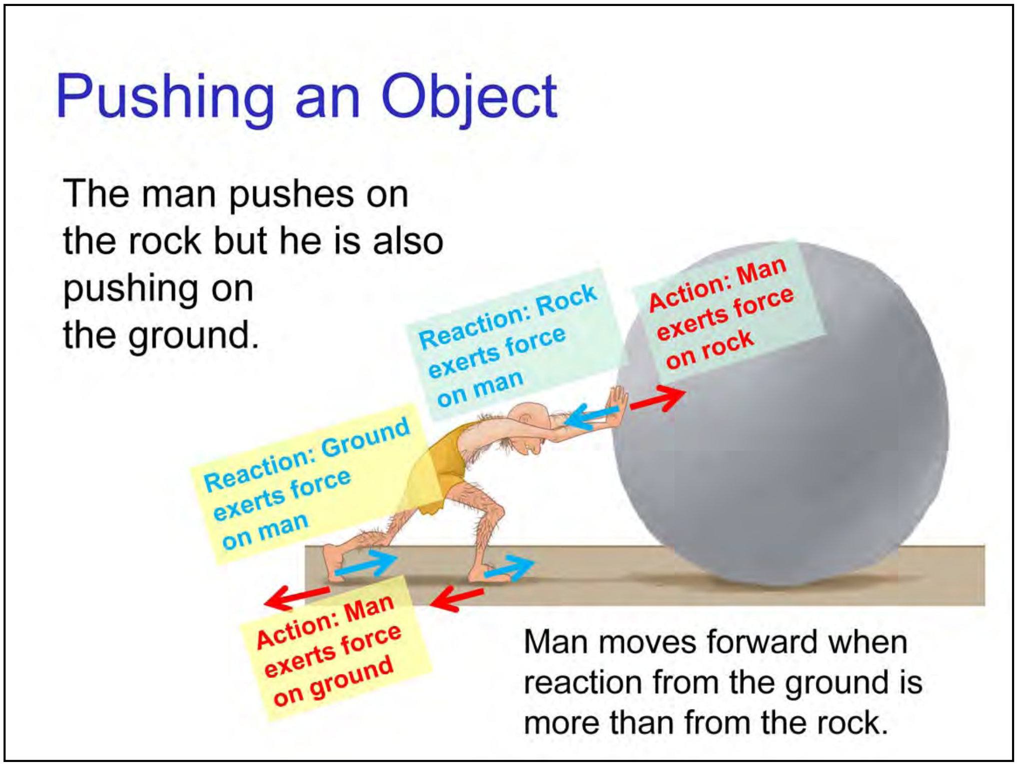
Illustrations by Dora Roychoudhury



Let's consider each part of this statement. First, the action and the reaction are a pair of matched forces. For example, suppose Mr. Alpha punches Mr. Beta. The action is the force of Mr. Alpha's fist hitting Mr. Beta's face so the reaction is the force of Mr. Beta's face pushing back on Mr. Alpha's fist. Note that it doesn't matter which force is labeled "action" and which one is "reaction"; you can always switch the names since they're symmetric. Second, these two forces are instantaneous so if Mr. Beta punches back that's not the reaction, that's a new action.

Furthermore, if Mr. Alpha punches with 100 pounds of force towards the left then there's an equal force of 100 pounds on his fist towards the right. Since forces change momentum the action force causes the Mr. Beta's head to start moving towards the left while the reaction force changes the momentum of Mr. Alpha's fist, possibly bringing it to a stop. To animate this scene successfully it's essential to match the action and reaction. A common mistake is to focus on animating Mr. Beta's motion while neglecting the reaction that must simultaneously be occurring on Mr. Alpha.

Illustration by Dora Roychoudhury



Judging the effect of action-reaction is complicated by the fact that there's usually several action forces to be considered. Take the simple case of a man pushing a rock. The man exerts a force on the rock so the rock exerts a force back on him. If he was on roller skates he'd move backwards and the rock would move forward. But he's barefoot so there's another set of action-reaction forces: he also pushes back on the ground and the ground pushes him forward. In order for him to push the rock forward the force exerted by his legs cannot be less than the force exerted by his arms. These actions and reactions have to appear to match in order to animate this scene believably.

Illustration by Dora Roychoudhury

Pulling a Character

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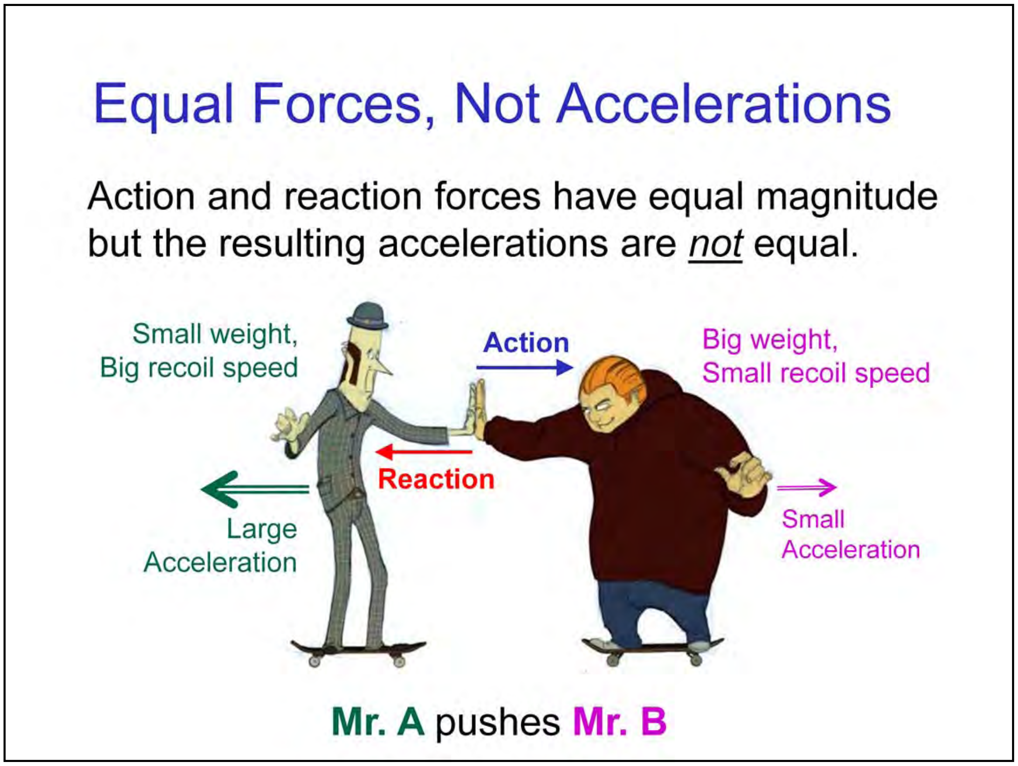
Pulling a Character

In this shot Gia is pulling Alex into the train car. Because she is exerting an action force on him there's an equally large reaction force pulling her towards the outside of the car (even if Alex is just passively holding his arm out to her). To stay in the train car, Gia pushes towards screen left with her feet; the resulting reaction force pushes her towards the inside of the car. If the two reaction forces on Gia plus the force of gravity (her weight) are in balance then her center of gravity won't move, otherwise she'll either be pulled out of the car or fall backwards into the car.



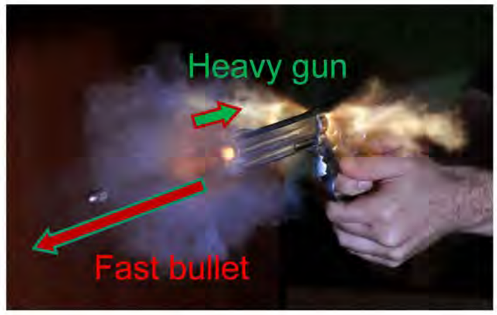
Although a pair of action-reaction forces have equal magnitude, they don't have equal effect. The resulting accelerations depend on the weight of the objects the forces are pushing. In this example, Mr. A (screen left) pushes on Mr. B; to make the resulting effect obvious we've put the on skateboards. The characters move apart due to the action-reaction forces but since Mr. A weighs less he will accelerate more, moving away at a high speed while Mr. B will move away more slowly due to his large weight. The difference between their recoil speeds indicates their relative weights.

Illustration by Rebbaz Royee

Recoil

Action/Reaction principle also describes recoil.

The action force that accelerates the bullet results in a matched reaction force in opposite direction, recoiling the gun.



Recoil Speed =
(Bullet/Gun Weight Ratio) x (Bullet Speed)

Action/Reaction principle also describes recoil. The action force that accelerates the bullet results in a matched reaction force in opposite direction, recoiling the gun. Specifically, when there are no other forces: Recoil Speed = (Bullet/Gun Weight Ratio) x (Bullet Speed).

http://commons.wikimedia.org/wiki/File:Bullet_coming_from_S%26W.jpg

Demo: Extinguisher Rocket

http://www.youtube.com/watch?v=9F_VjHZATbY

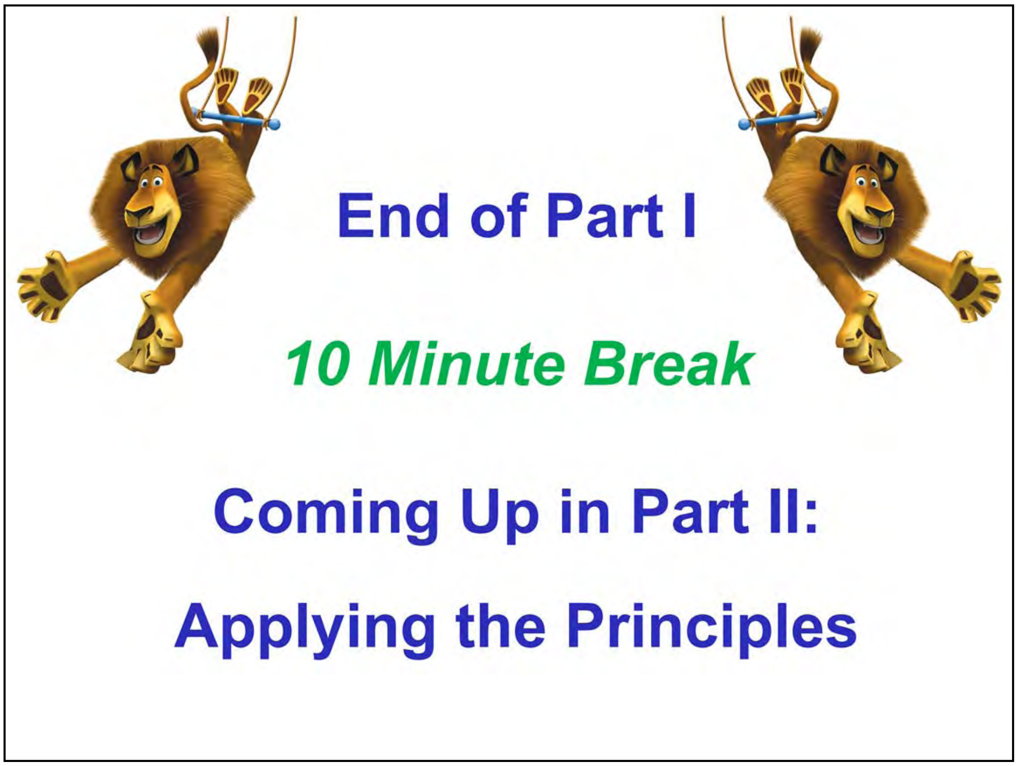
Using a fire-extinguisher for rocket-like propulsion.

Demonstration of recoil using a fire-extinguisher as rocket-like propulsion. Because the force is small the demonstration is performed standing on a swivel platform.

Manipulating Action-Reaction

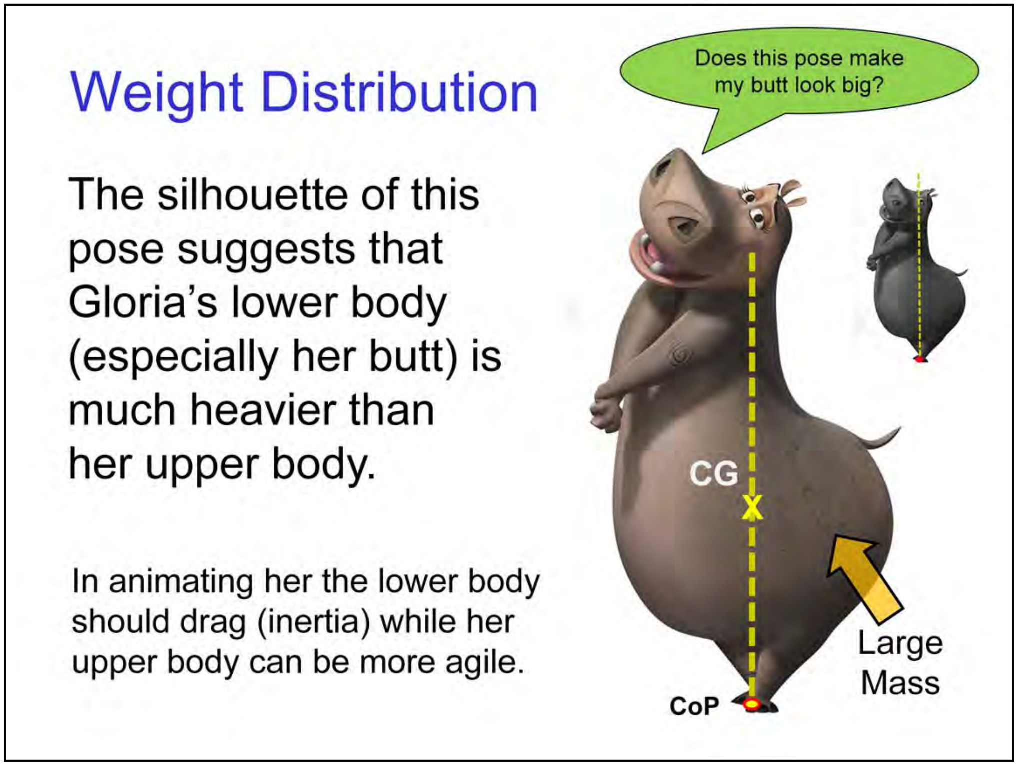
Action/Reaction principle is often manipulated to give "weight" to a dramatic character or to make a comedic character "light."

Finally, the action-reaction principle is often violated in films, both animated and live-action. Heroes deliver powerful punches (or shoot big guns) with negligible recoil while the villain goes flying backwards. This may be done intentionally for dramatic effect (to make the hero look powerful) or for comic effect (to defuse the violence) but if it's unintentional and unexpected then there's a chance of shattering the audience's suspension of disbelief.


End of Part I
10 Minute Break
Coming Up in Part II:
Applying the Principles

Part II: Applying the Principles

Basic Poses



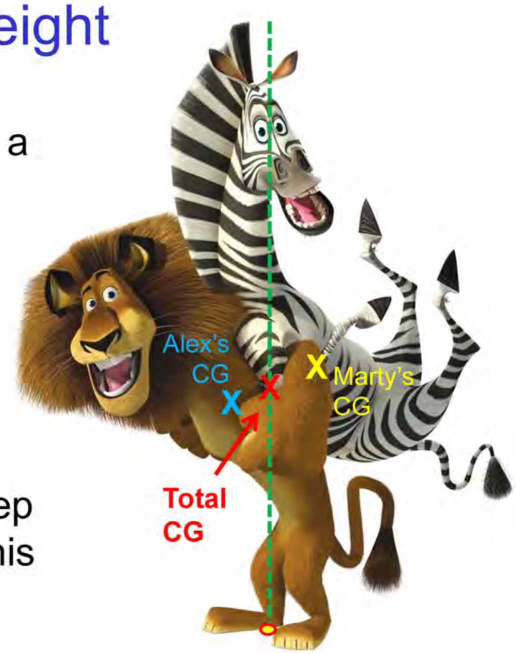
The principle of the Center of Gravity tells us that in a stationary pose the character's center of gravity is located over its base of support. Recall that to check this it's helpful to draw the “line of gravity”, the vertical line passing through the center of gravity, and the “center of pressure”, which is where the line of gravity hits the floor. If the center of pressure falls inside the base of support then the force of gravity is balanced by the upward force of the floor.

A character’s pose tells us something about the character’s weight distribution. If the weight is evenly distributed then the center of gravity is close to the visual, geometric center of the body. But if the weight is not evenly distributed then the pose will be asymmetric.

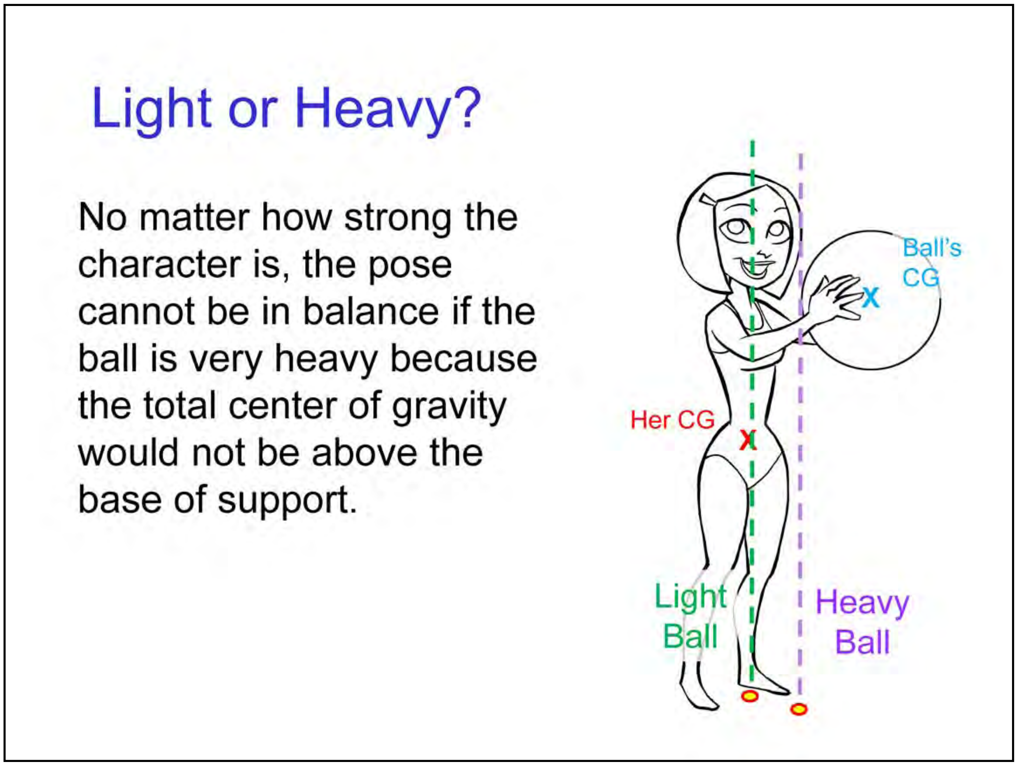
Carrying a Weight

A heavy weight on a character's back shifts the total center of gravity (CG) towards the rear.

The character will lean forward to keep the total CG over his Base of Support.

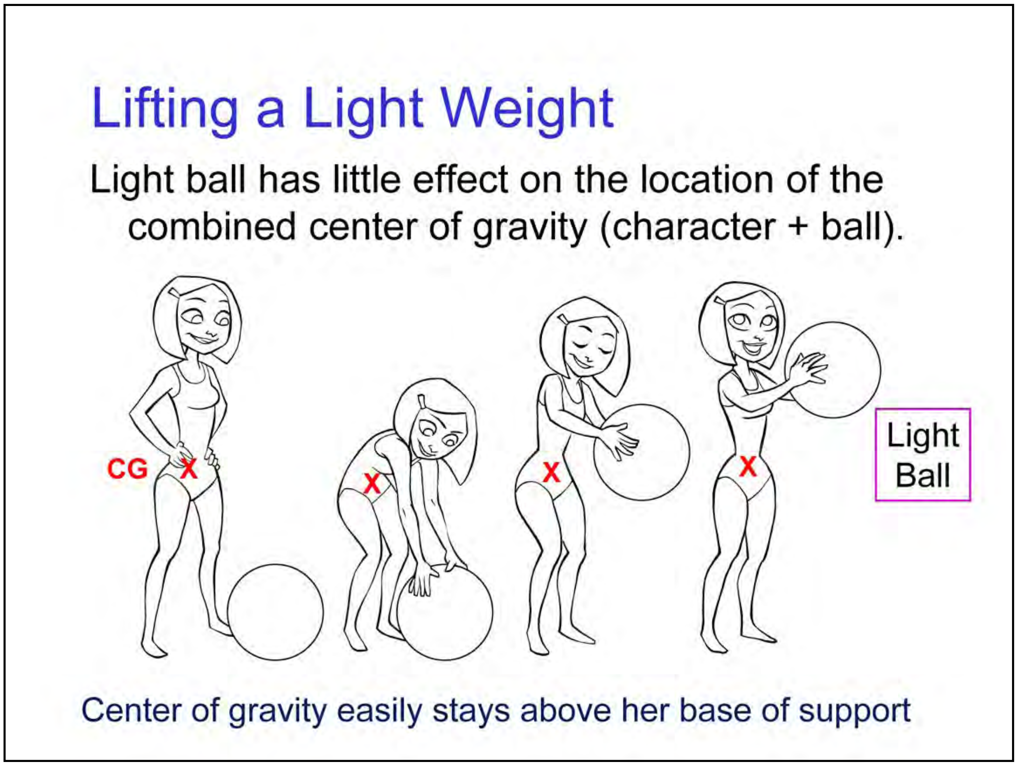


A character has to lean forward when carrying a heavy weight on his back in order to keep the combined center of gravity above the base of support. In this image we can visually estimate the location of Alex and Marty's individual centers of gravity. The combined total CG will be located between the individual CGs and will be proportionally closer to the heavier character.



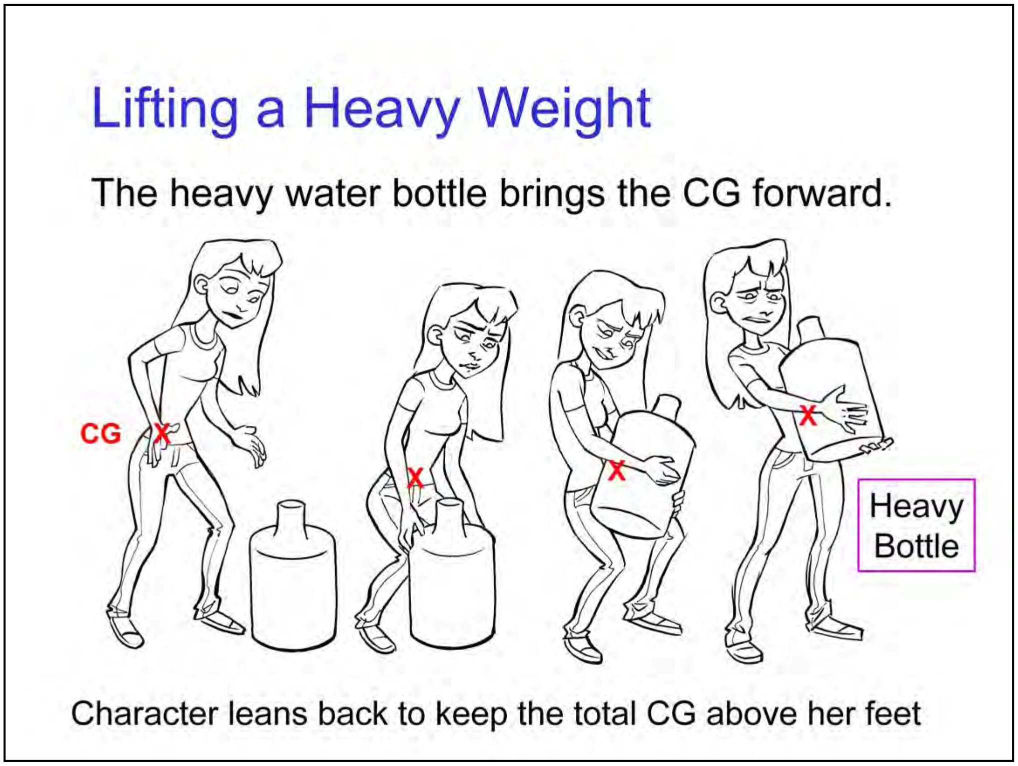
This character cannot be carrying a heavy ball because, regardless of her strength, if the ball were actually very heavy then the combined center of gravity would be in front of her, outside her base of support. Imagine this as a rigid, free-standing statue; it would tip forward if the ball was very heavy.

Illustration by Charlene Fleming



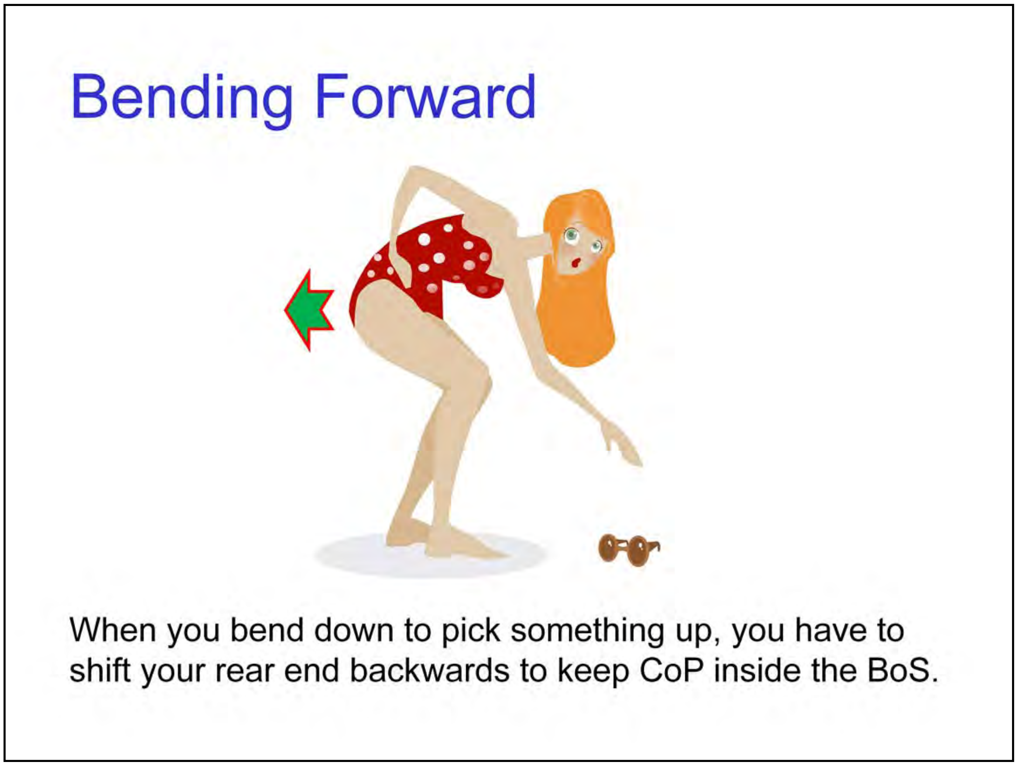
Notice the difference in the poses for characters lifting light and heavy objects. In lifting a light ball, say a beach ball, the position of the ball has little effect on the combined center of gravity.

Illustration by Charlene Fleming



The character lifting the heavy object has to keep it close to her body to position the combined center of gravity over her feet. If she lifts it quickly (accelerating it upward) then it's even harder for her to stay in balance since the object effectively has greater weight (see the principle of Weight Gain and Loss), which shifts the combined center of gravity.

Illustration by Charlene Fleming

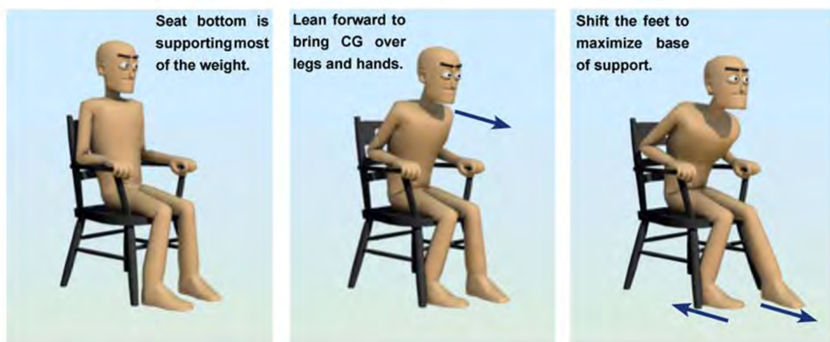


The character picking up her sunglasses has to move her hips backwards as she leans forward in order to maintain her center of gravity over her feet.

Illustration by Katie Corna

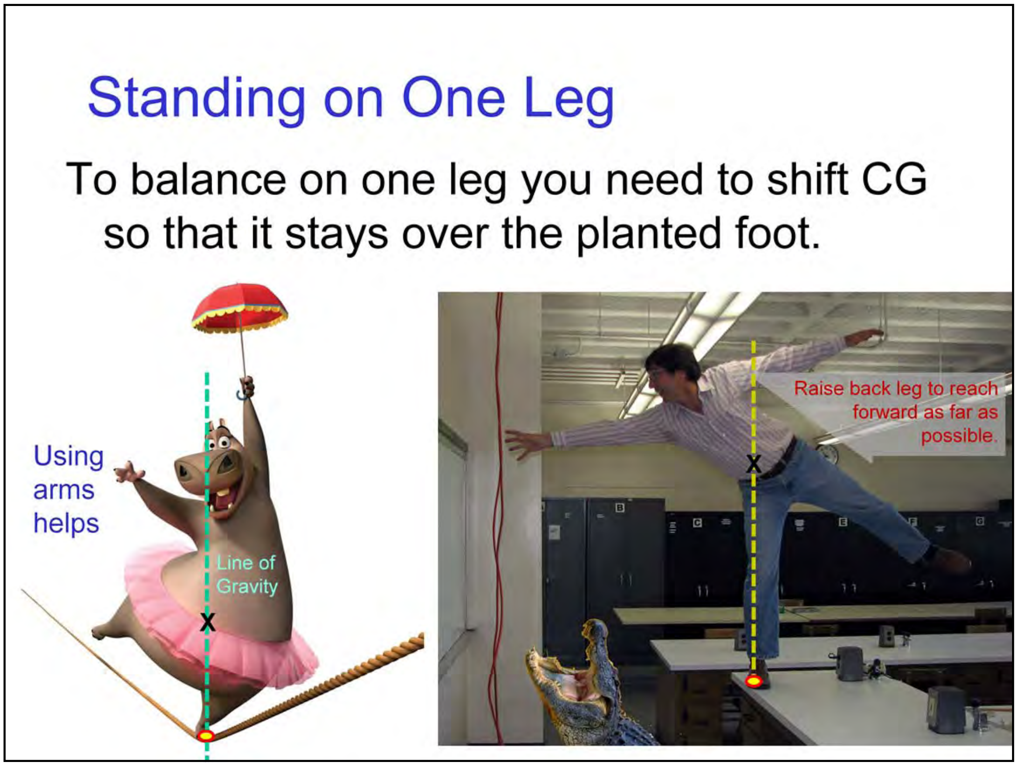
Rising from an Armchair

To rise from a chair, you need to lean forward to bring your center of gravity over your feet.

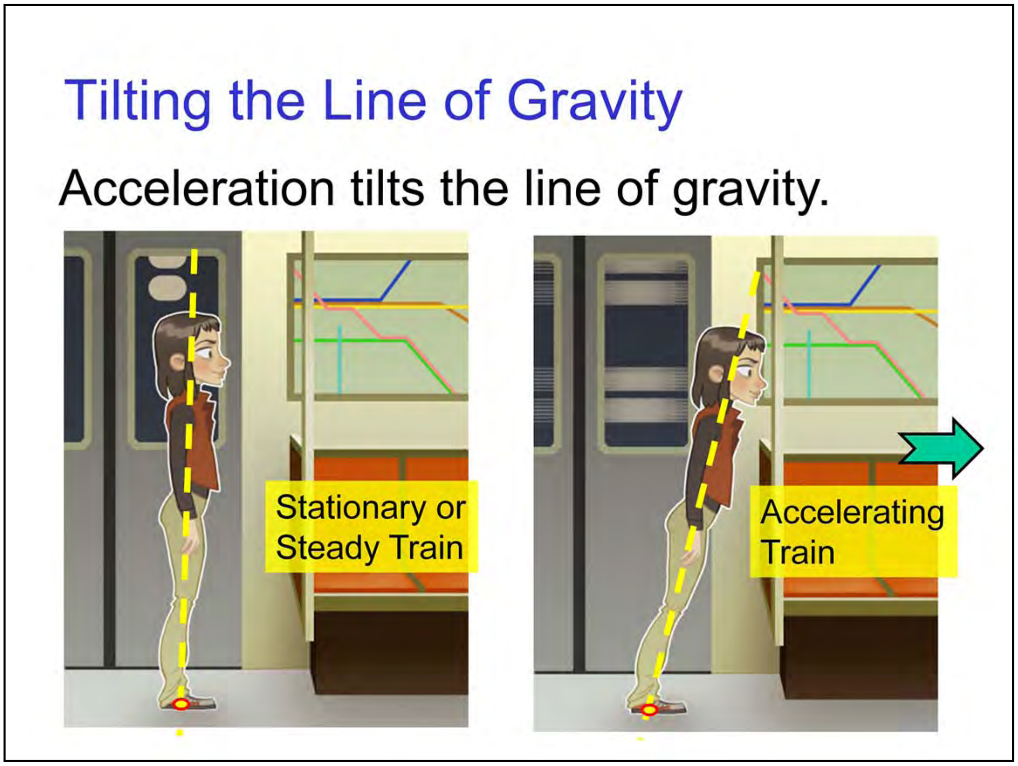


The character rising from a chair leans forward to position his center of gravity over his feet; he also shifts his feet to broaden his base of support.

Illustration by Corey Tom

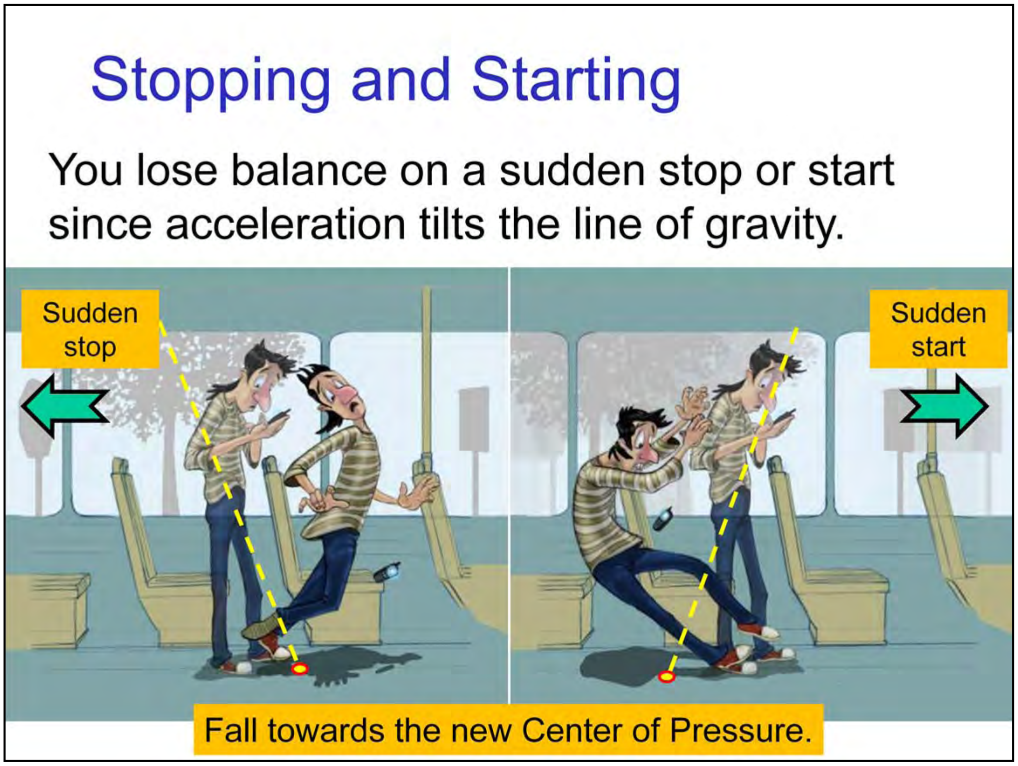


Standing on one leg is difficult because the base of support is small. In general, when the base of support is narrow we instinctively raise our arms to our sides since moving them allows us to rapidly reposition our center of gravity. When reaching over an edge, as in the photo, to keep the center of gravity over the foot the other leg is extended as far back as possible.



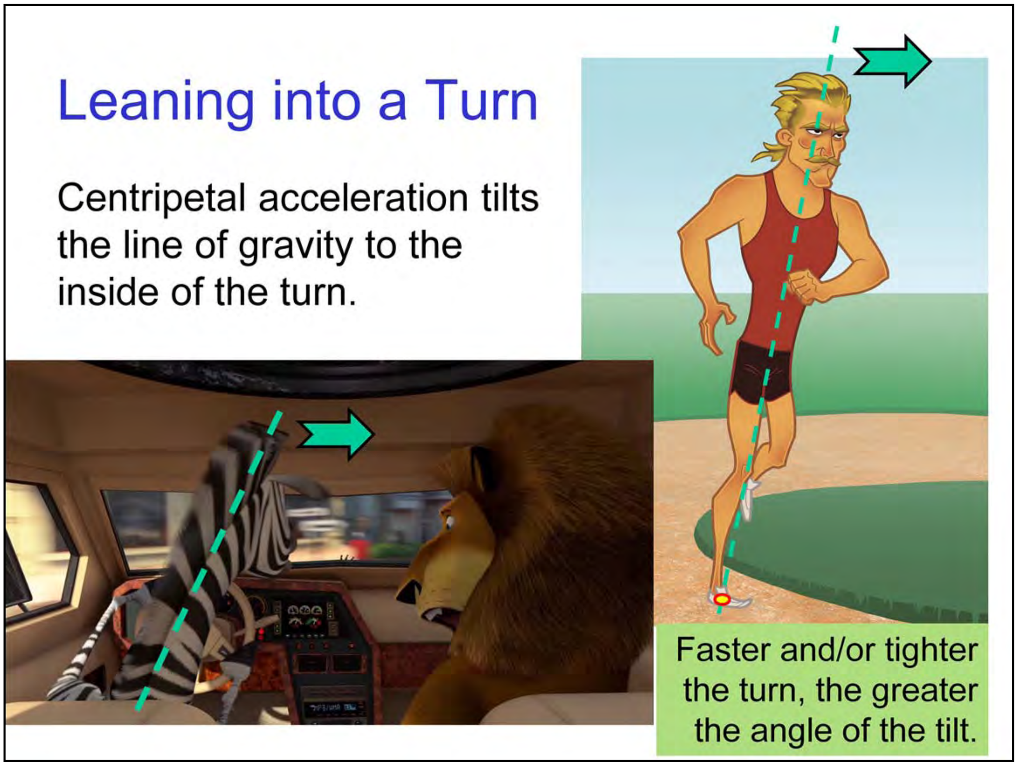
In exceptional cases the line of gravity is not vertical. Consider a woman standing in the aisle of a train. As the train accelerates the line of gravity tilts forward so the woman leans forward to stay in balance.

Illustration by Charlene Fleming



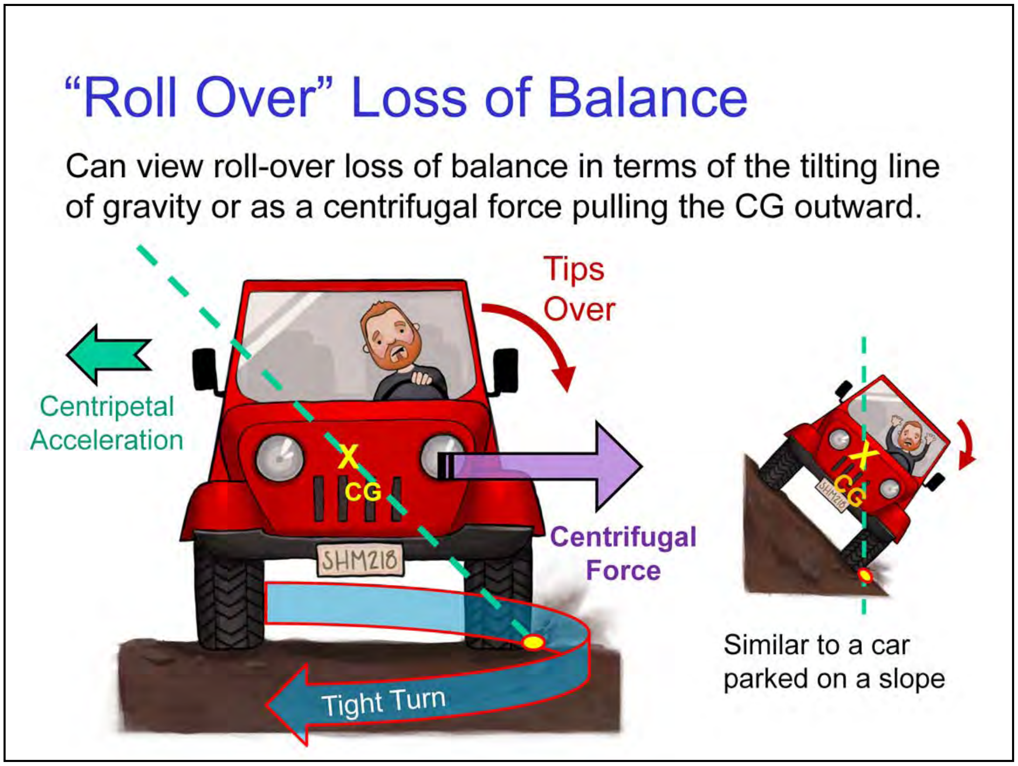
The distracted bus passenger is out of balance when the bus comes to a sudden stop or starts moving suddenly. Acceleration tilts the line of gravity and moves the center of pressure out from under his base of support. Once the bus is moving with a uniform speed the line of gravity returns to being vertical.

Illustration by Rebbaz Royee



When a moving character turns a corner there's a change of momentum due to a centripetal force (see the principle of Momentum and Force). Remember that a change in momentum occurs when the direction of motion changes, even if the speed stays constant. Making a turn causes the line of gravity to tilt and characters will lean accordingly to stay in dynamic balance (e.g., a runner leans into a turn). The faster and/or tighter the turn, the greater the angle of the tilt.

Illustration by Charlene Fleming



From the point of view of a spectator watching the car take the turn there’s a centripetal force (and acceleration) as the car’s momentum changes from traveling towards screen right to going towards screen left. This acceleration tilts the line of gravity and if the center of pressure goes outside the car’s base of support (perimeter around the four tires) then the car loses balance and tips over.

From the point of view of the driver, it’s as if a centrifugal force is pulling the car towards screen right; if that force is large enough then the car lose balance and roll over.

Illustration by Stephanie Lew

Leaning In vs. Tilting Outward

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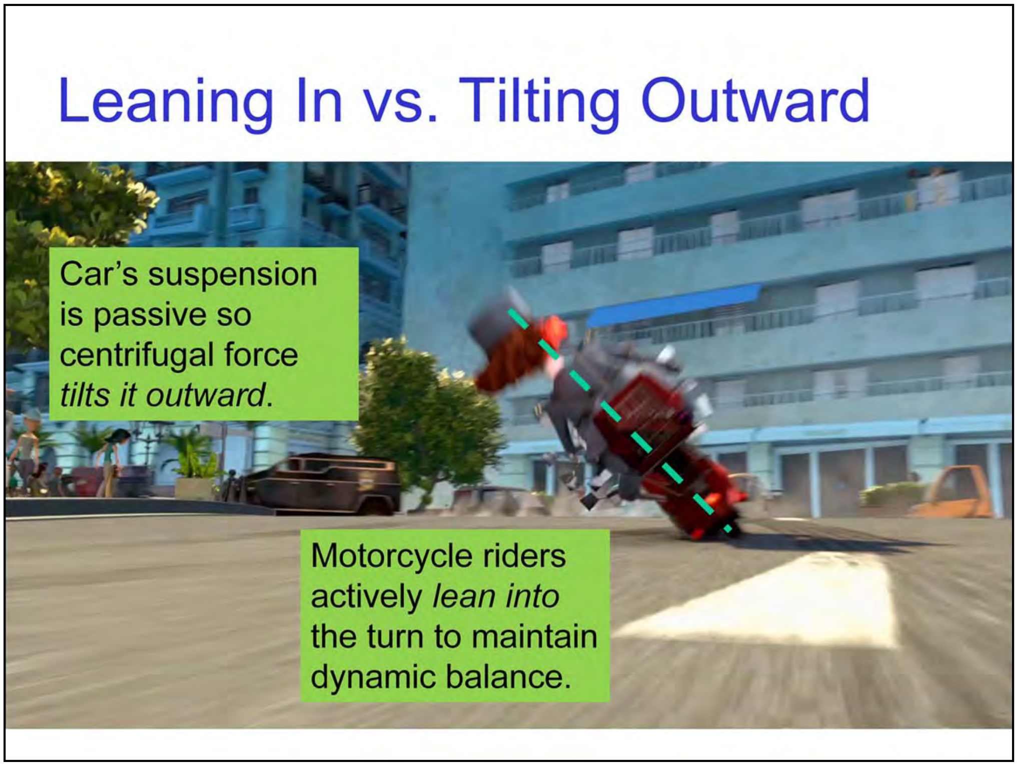


When a car takes a high speed turn by the Law of Inertia the body of the car tends to continue traveling in the original direction, thus causing the car to tilt to the outside of the curve; this can also be thought of in terms of the centrifugal force, as experienced by the car.

On the other hand, motorcycle riders tend to lean into the turn thus tilting opposite from the tilting of the car, in order to maintain dynamic balance as they take the turn.

Part II: Applying the Principles

Jumps

Center of Gravity in Jumps

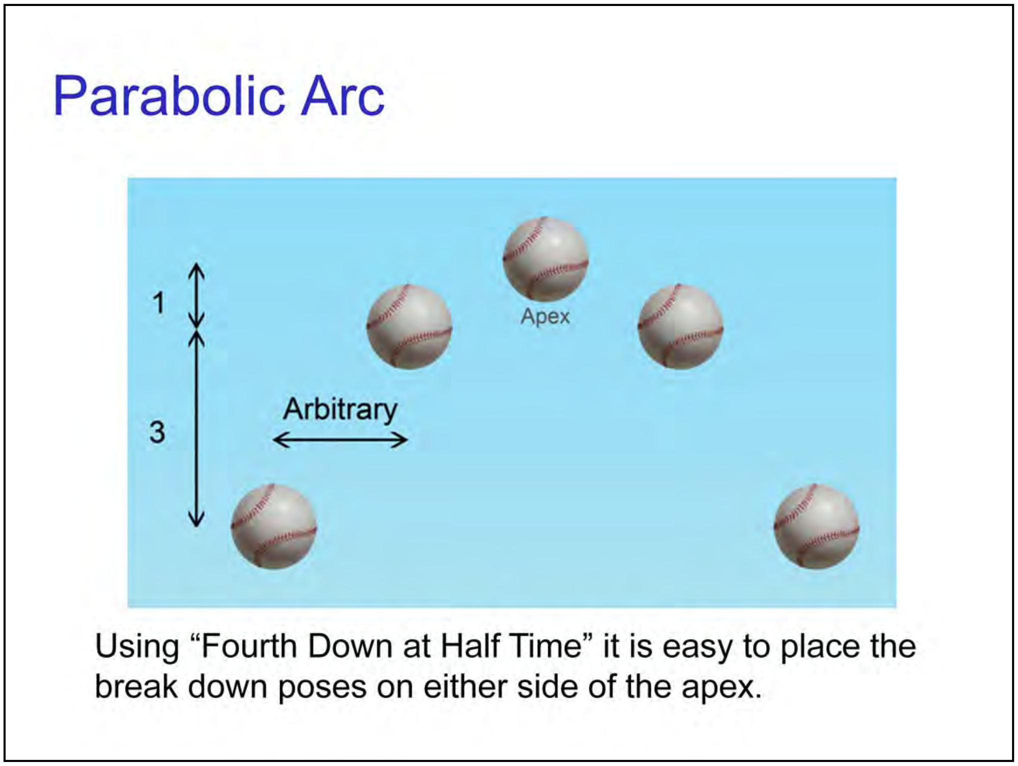
Corey Tom

http://www.youtube.com/watch?v=_UHMbrfVyGc

For a jumping character the path of action for the center of gravity is a parabolic arc.

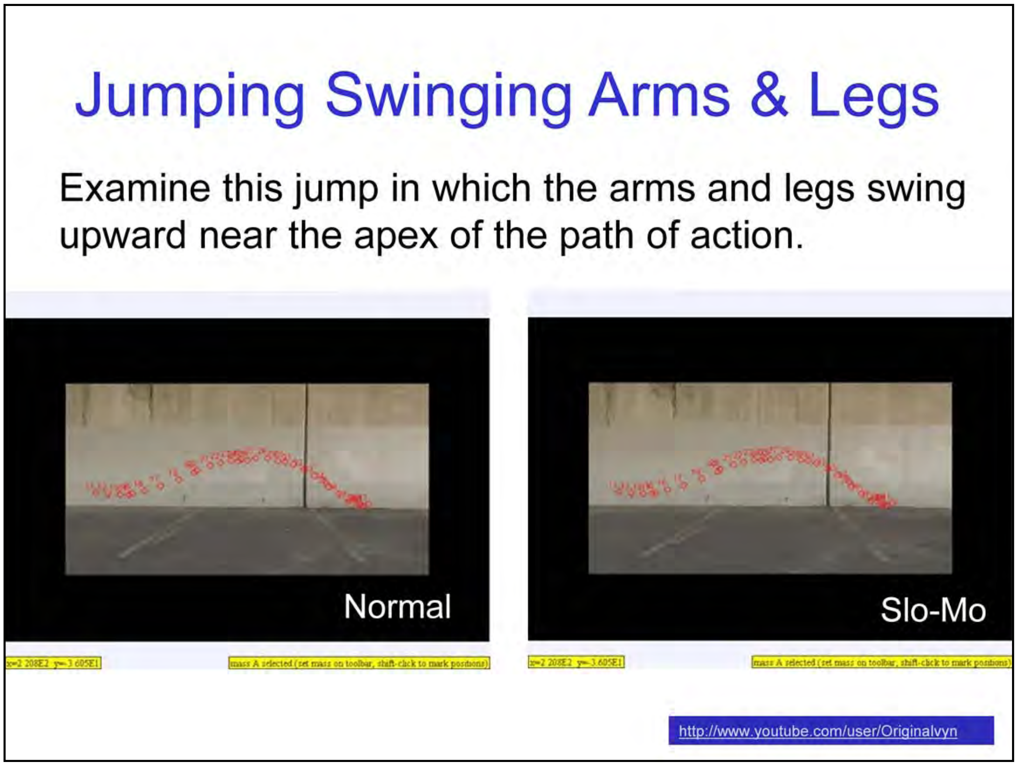
Next, let’s apply the principles to the analysis of a character jump. We’ll divide a jump into three phases: The “push” phase when the character goes from the lowest point of the crouch until take-off. The “jump” phase when the character is in the air; for convenience we’ll consider this in two parts, namely rising to the apex and falling from the apex. Finally, the “landing” phase is from touch-down to the completion of the settle.

We'll start with the easiest part of a jump, which is when the character is in the air. For human-sized characters the force of air resistance is small when traveling below 30 miles per hour so typically the only significant force during a jump is gravity. In that case the path of action is a parabolic arc.



In a parabolic arc the horizontal spacings are uniform and the vertical spacings are given by the Odd Rule and Fourth Down at Half Time (see the principle of Timing, Spacing, and Scale). In the rare cases where air resistance is significant the path of action is skewed, with the descent being at a steeper angle than the ascent.

Illustration by Charlene Fleming

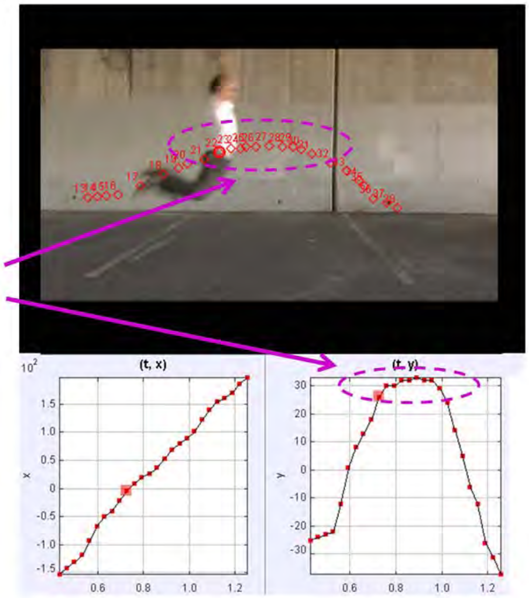


It is important to remember that the center of gravity follows a parabolic arc but it is important to remember that the center of gravity shifts when a person moves their arms and legs. In this video the center of the waistline is tracked and this path of action is flatter than a parabolic arc near the apex.

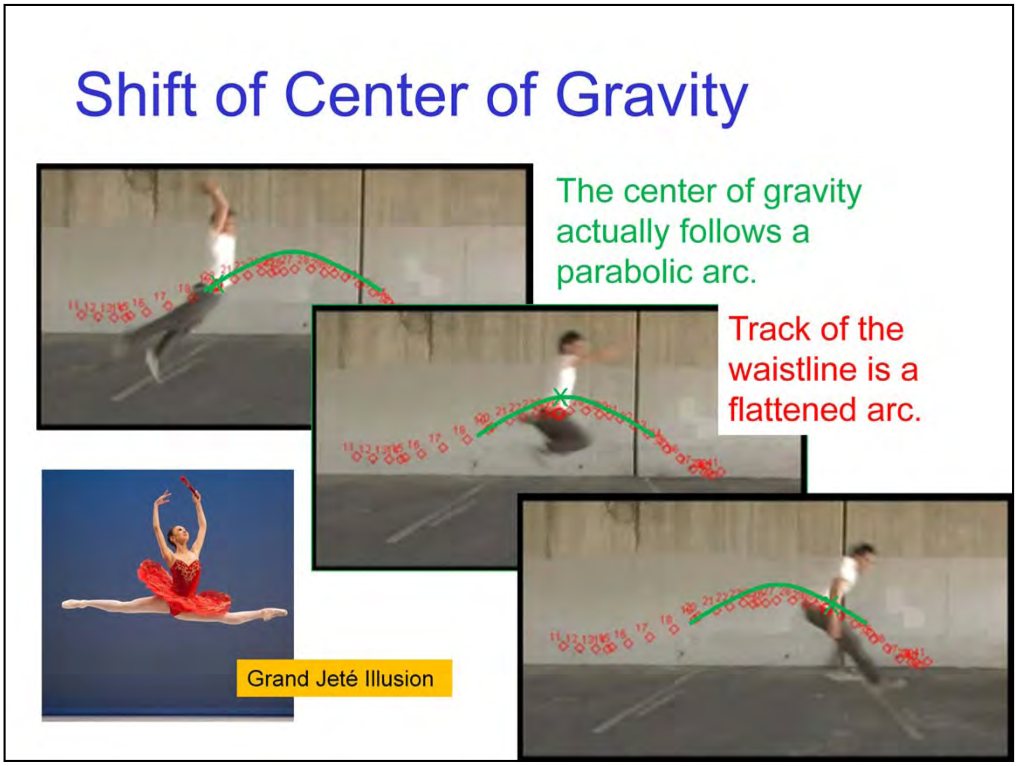
Flattened Arc

The arc around the apex is flattened, giving the illusion of floating or flying.

Due primarily to the lifting of the legs in this jump.

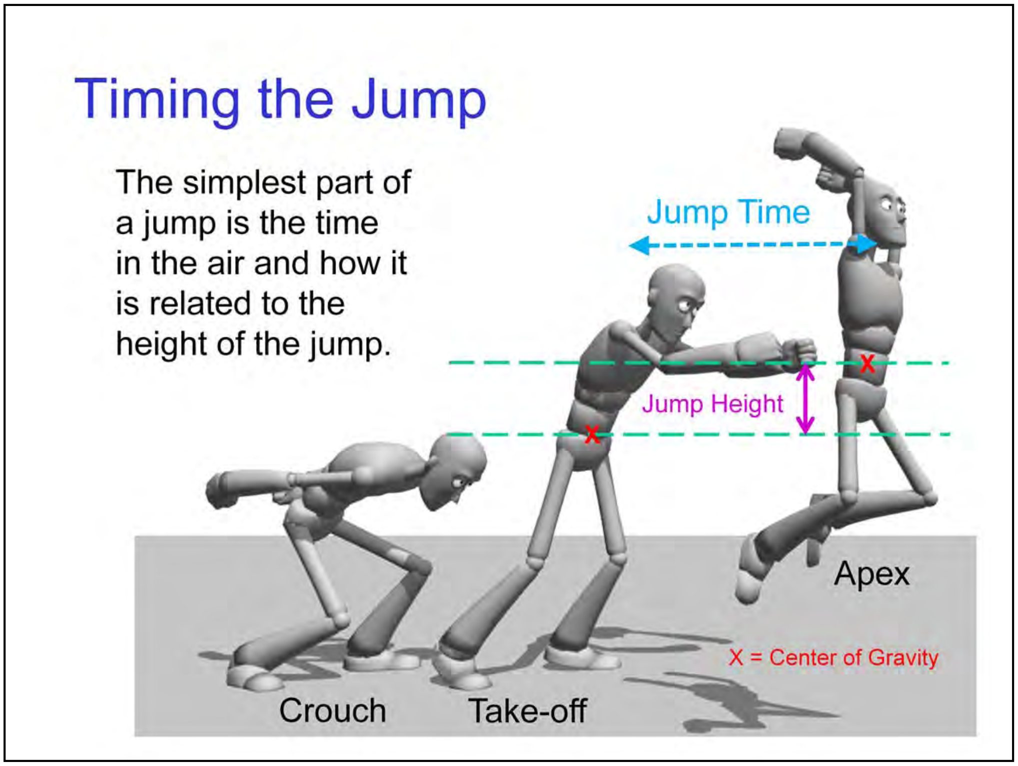


These graphs of the horizontal and vertical position of the waistline clearly show the flattening of the path of action for the center of the waistline.



The center of gravity actually follows a parabolic arc, independent of the motion of the arms and legs. However the position of the center of gravity shifts location so when the character raises his legs around the apex of the jump the center of gravity shifts above the waistline. This effect is known as the Grand Jeté Illusion in which a ballet dancer appears to float around the apex of their jump; this illusion is achieved by raising the arms and legs as the dancer enters the apex of the jump and lowering them on the descent.

Ballet photo: http://commons.wikimedia.org/wiki/File:Chenxin_Liu_-_Don_Quichotte,_Kitri_-_Prix_de_Lausanne_2010-7_edit.jpg



The timing of the character while in the air is also simple since gravitational acceleration is the same for all objects. That is, from the height of the jump (vertical distance from take-off to apex) we can find the jump time (number of frames from take-off to apex).

Image by Corey Tom

Jump Time & Height

The same table we saw for the ball drop gives the jump time (from take-off to apex) and jump height.

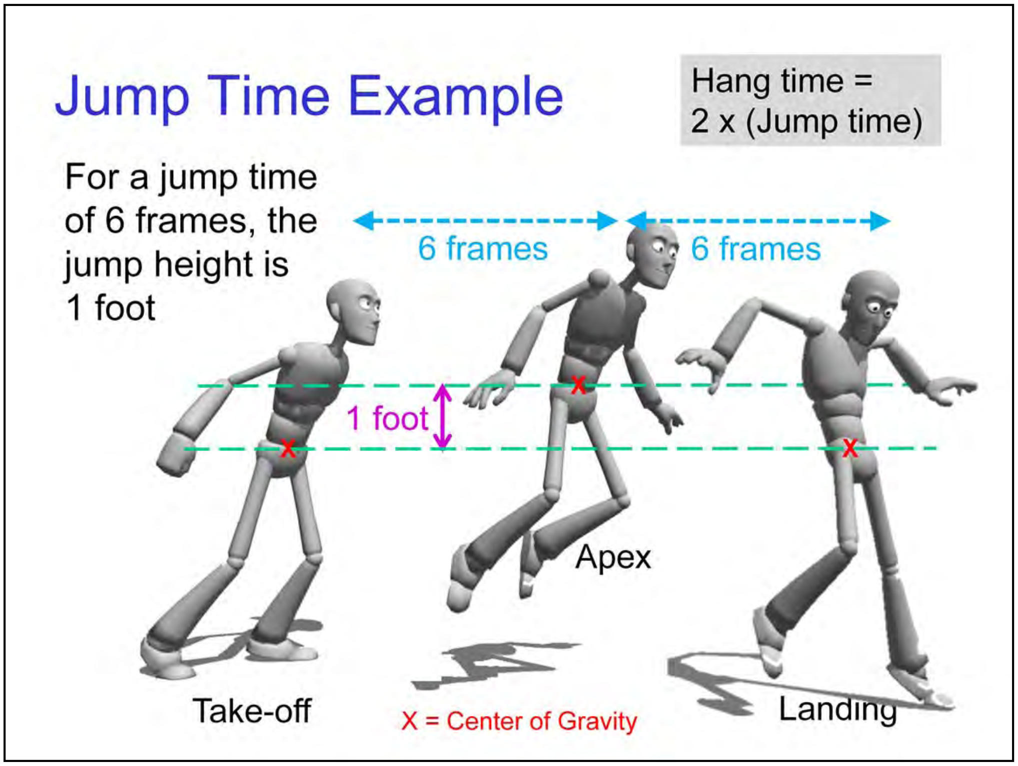
The formula to compute this table is:
(Distance in inches) =
(Number of Frames) x
(Number of Frames) x
(1/3 inch)

Jump Time (seconds)	Frames	Jump Height
$^{1}/_{24}$	1	$^{1}/_{3}$ inch
$^{1}/_{12}$	2	1 $^{1}/_{3}$ inches
$^{1}/_{8}$	3	3 inches
$^{1}/_{6}$	4	5 $^{1}/_{3}$ inches
¼	6	12 inches
$^{1}/_{3}$	8	21 inches
½	12	4 feet
$^{2}/_{3}$	16	7 feet
¾	18	9 feet
1	24	16 feet

This table lists the jump heights for various jump times. The values in the table are obtained using the formula:

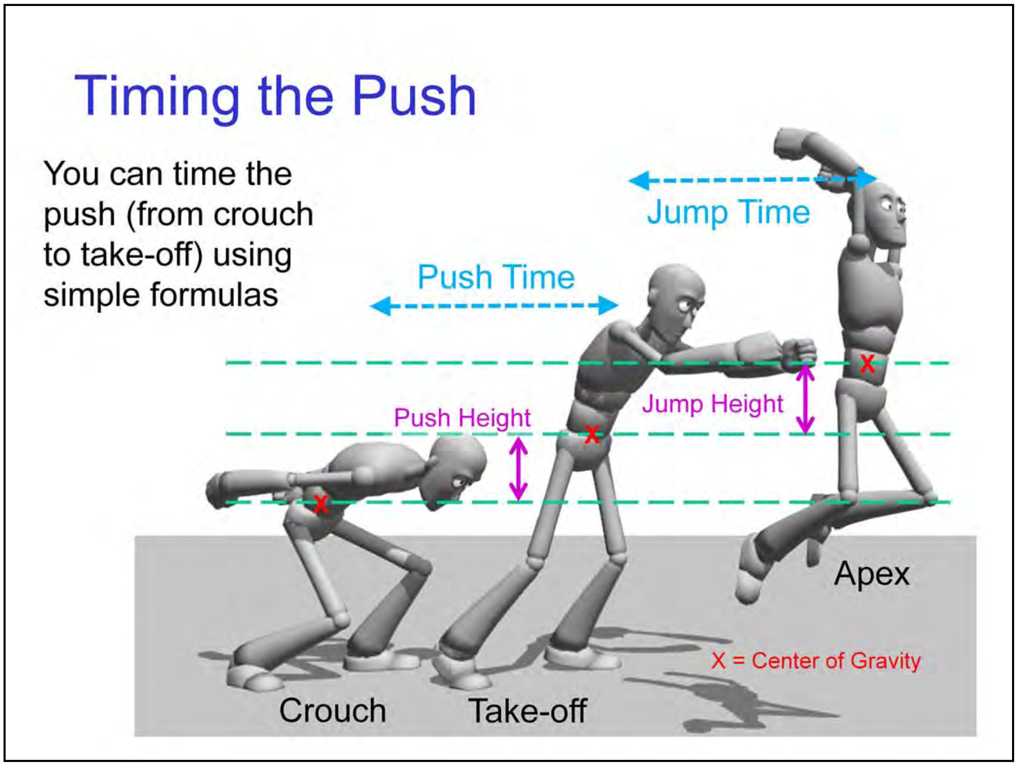
(Jump height in inches) = (1/3 inch) x (Jump time in frames) x (Jump time in frames)

For example, for a jump time of 6 frames the jump height is (1/3) x (6) x (6) = 12 inches



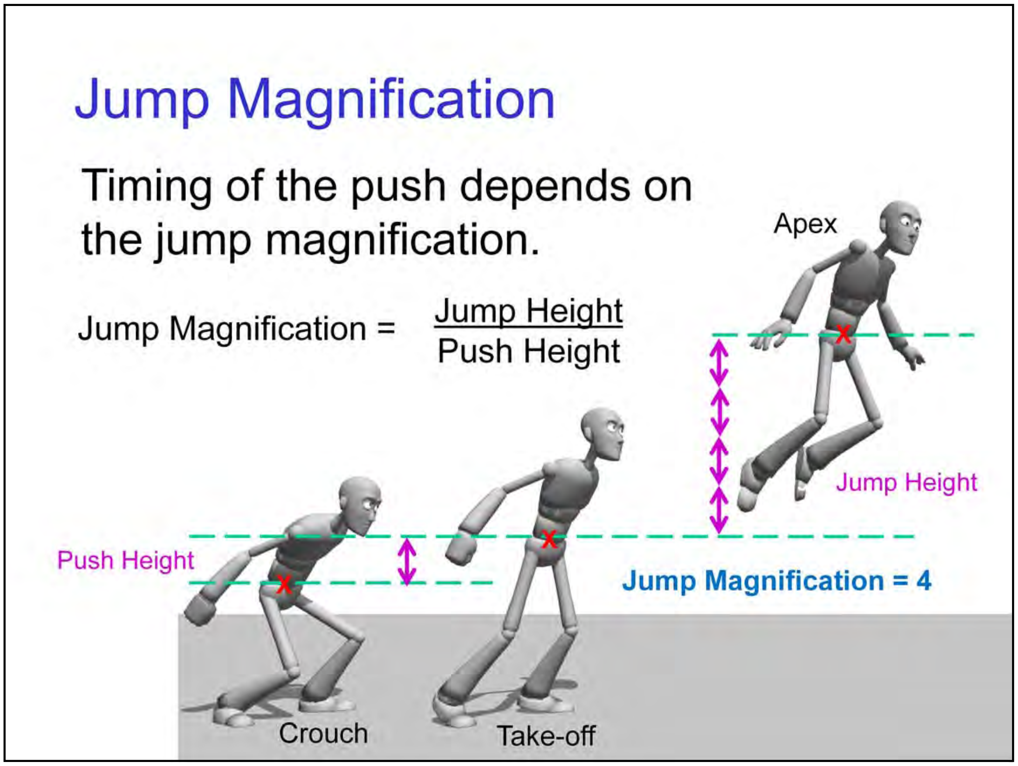
For example, everything falls one foot in six frames so jumping one foot in height takes six frames from take take-off to apex and another six frames from apex to landing. This corresponds to a “jump height” of one foot and a “jump time” of six frames. Timing and scale are connected so a four foot jump takes twice as long. Notice that the timing only depends on the (vertical) height of the jump, not on the (horizontal) distance jumped.

Image by Corey Tom



Now let's consider the start of a jump, when the character is rising out of the crouch, pushing with its legs in order to get into the air. We'll call the distance over which the character pushes off the “push height” and the distance from take-off to apex the “jump height.” Similarly, the time from crouch to take-off we’ll call the “push time” and from take-off to apex we’ll call the “jump time.”

Image by Corey Tom



The ratio of the jump to push heights is the ``jump magnification",

Jump Magnification = (Jump height, from take-off to apex) / (Push height, from crouch to take-off)

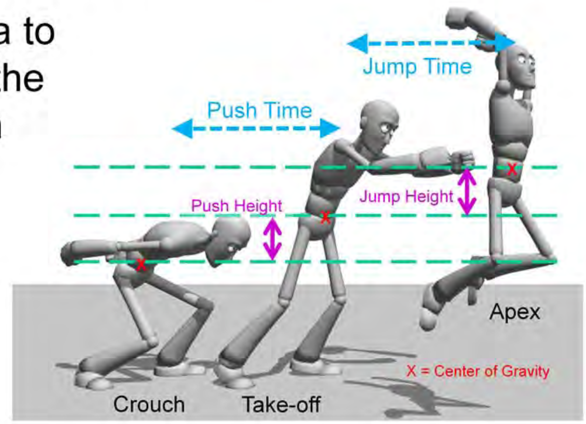
For example if the character rises from a one foot crouch and jumps four feet into the air then the jump magnification is 4.

Image by Corey Tom

Formula for Timing the Push

$$\text{Push Time} = \frac{\text{Jump Time}}{\text{Jump Magnification}}$$

Can use this formula to check the timing of the push compared with the jump time.

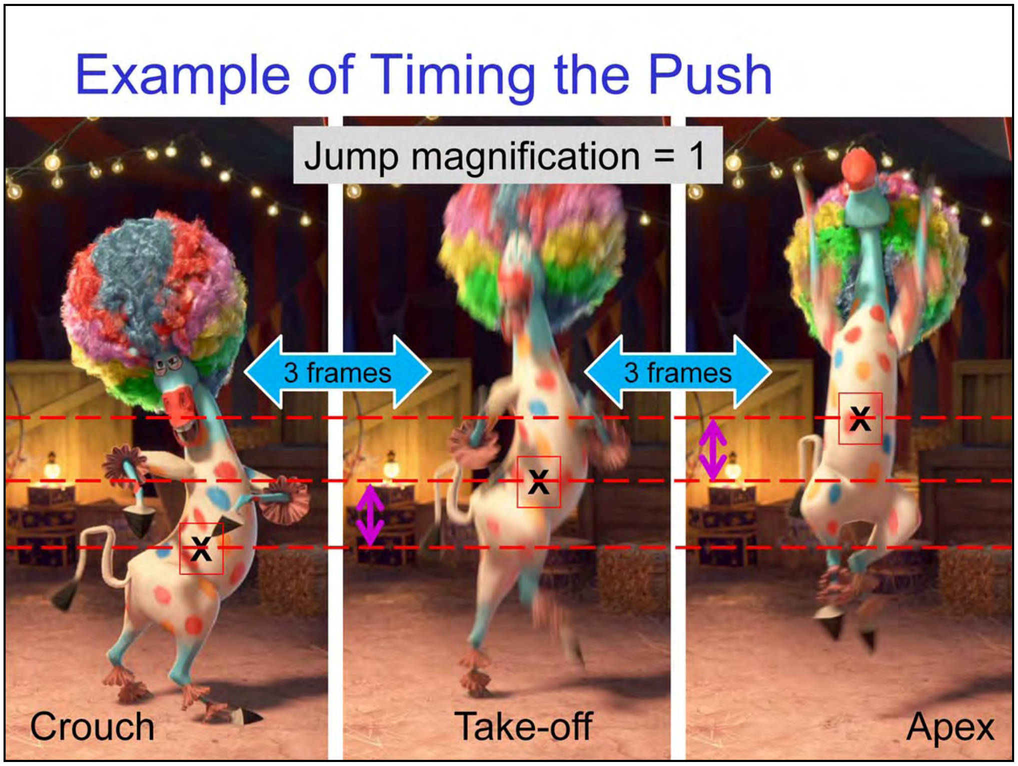


The reason that the jump magnification is important is that it determines the timing of the push. The larger the jump magnification, the quicker the push needs to be in order for the jump to look believable. Specifically, if the character pushes with constant force then,

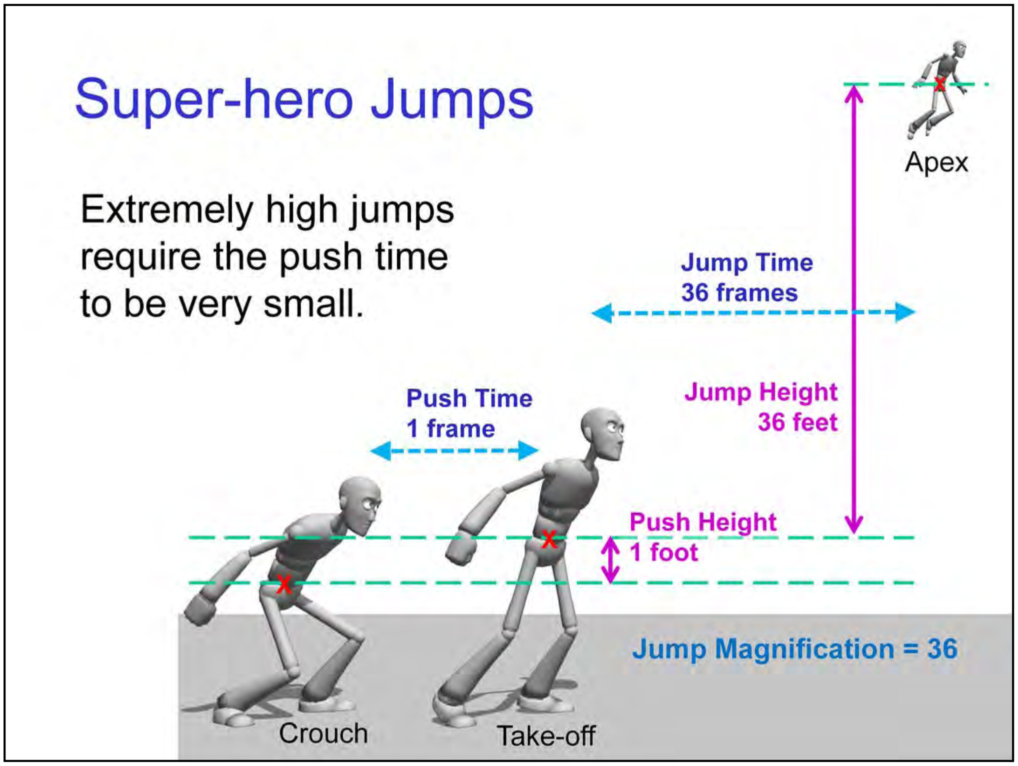
Push time, from crouch to take-off = (Jump time, from take-off to apex) / (Jump Magnification)

Image by Corey Tom

Example of Timing the Push



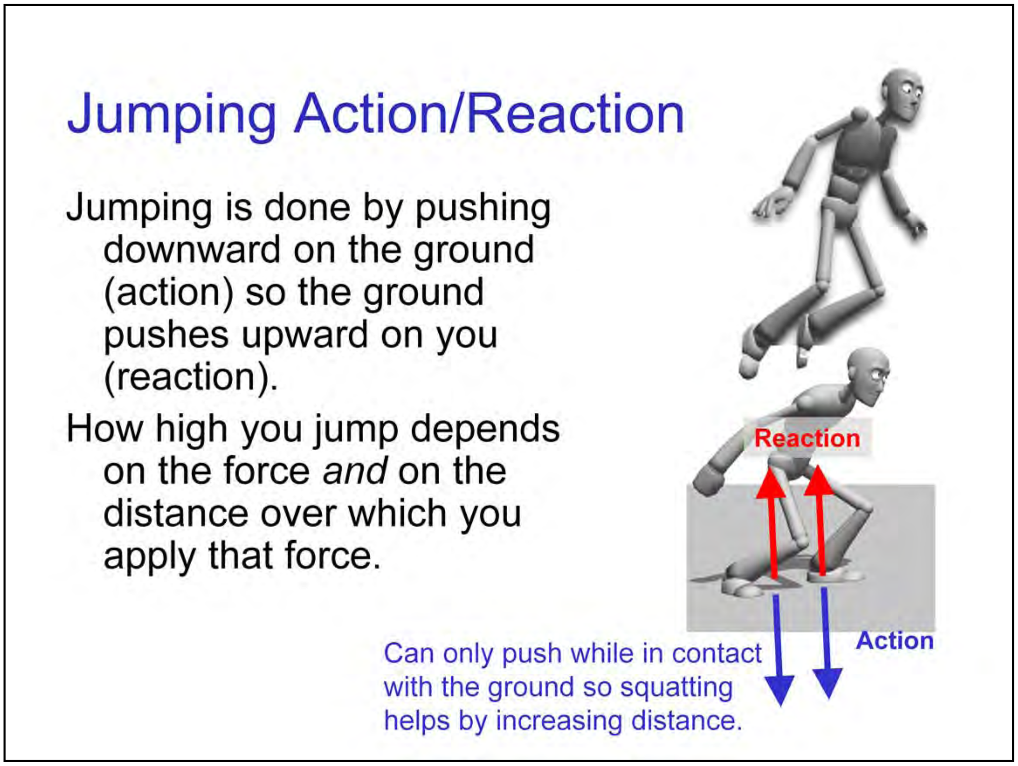
In this simple example the jump magnification equals one since the push height (crouch to take-off) equals the jump height (take-off to apex). That means that the push time (number of frames from crouch to take-off) equals the jump time (number of frames from take-off to apex). Notice that the jump time is actually a little short given the height of the jump, nevertheless the jump is believable because the timing is consistent with the jump magnification.



This is a more complicated example in which the character's jump height is about 36 feet, which translates to a jump time of 36 frames. Since the jump magnification is x36 the jump time is x36 the push time, which translates to a push time of only one frame.

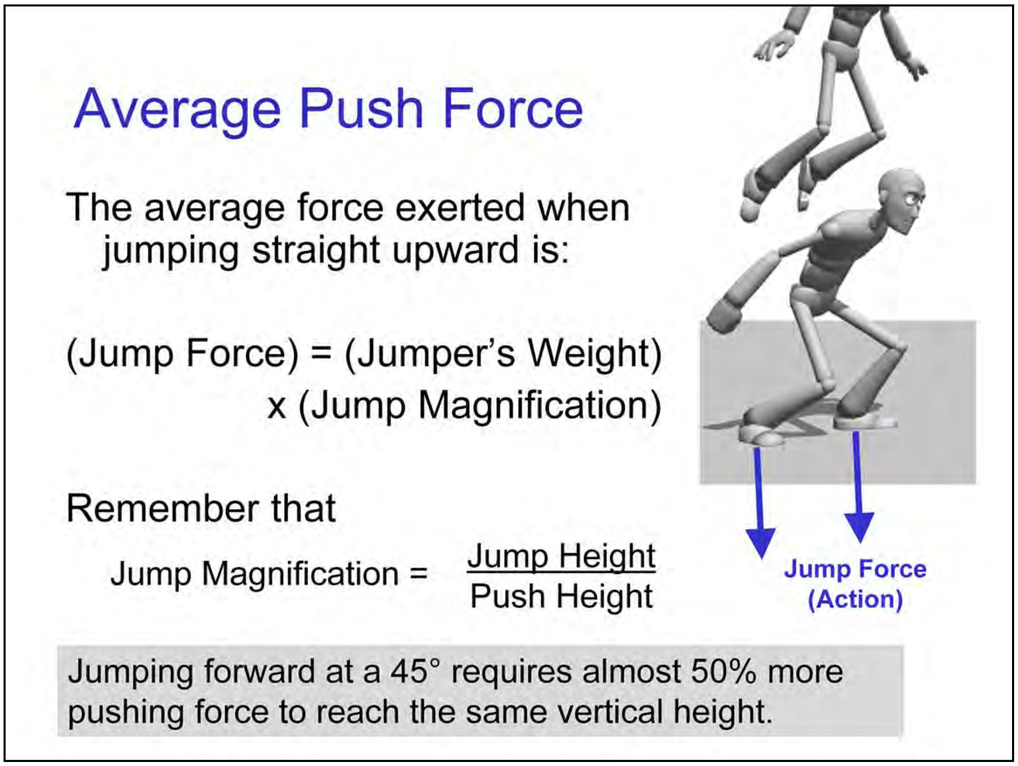
This calculation is not as important as the concept that it illustrates: The timing when a character is pushing off the ground needs to be quick if the jump is high. When a powerful super-hero jumps exceptionally high the physically accurate push time could be less than one frame. This is why such scenes are sometimes presented as occurring in slow-motion.

Image by Corey Tom



With great force comes great momentum change (and vice versa). The character exerts an action force down on the floor and so it's the reaction force that actually accelerates the character upward. Normally the ground doesn't move but if the character is jumping out of a small boat or from a tree limb then there is recoil (see the principle of Action-Reaction).

Image by Corey Tom



When jumping straight upward the average force that a character exerts during the push equals the jump magnification times the character's weight. Obviously, extremely high jumps require that the character exert a very large force, especially if the character is heavy. If the character also jumps forward as well as upward then the force is even greater, for the same vertical height. For example jumping forward at a 45 degree angle requires almost 50% more force.

Image by Corey Tom

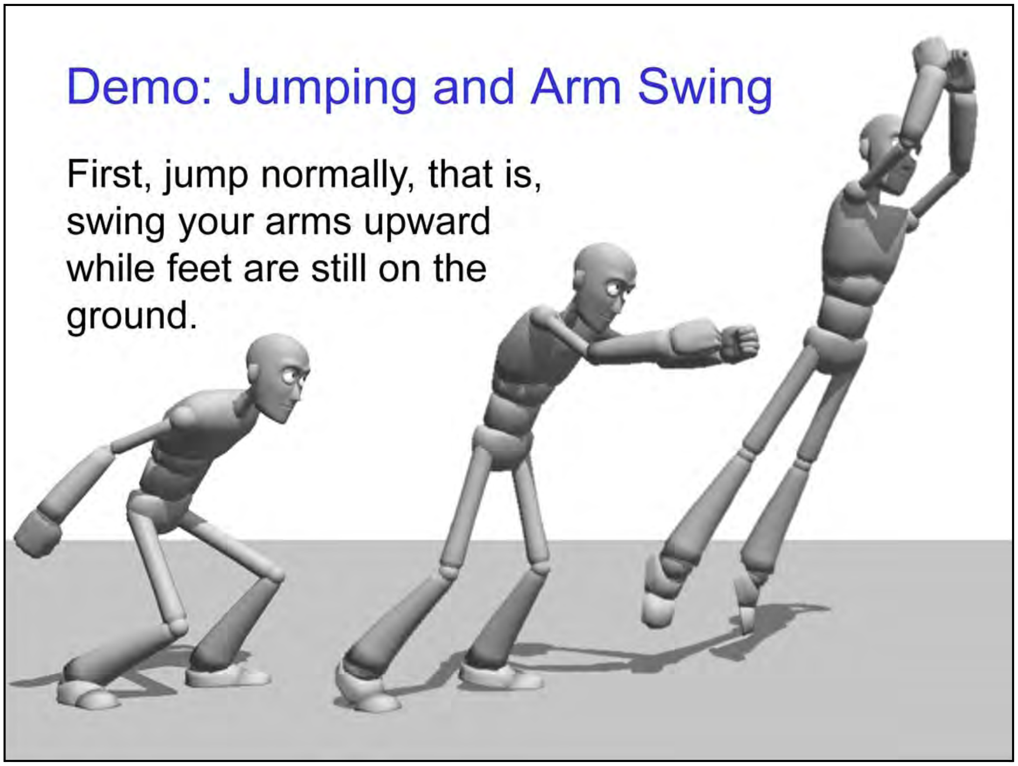
Swinging Arms in a Jump

The natural motion when jumping is to swing the arms upward as fast as possible *while* the feet are in contact with the ground.

Swinging the arms raises the center of gravity and increases the downward action force pushing off the ground.

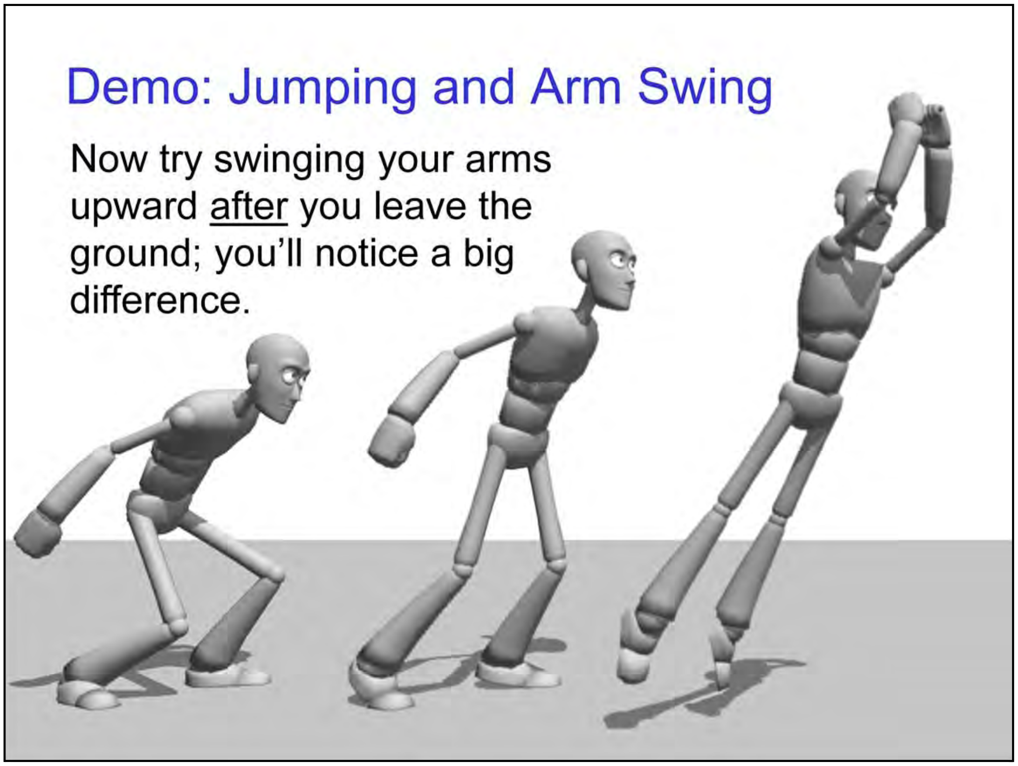
The reaction force from the ground that accelerates the character upward increases significantly (and the jump is higher) if the character swings the arms upward while pushing off on the ground.

Photo:
http://commons.wikimedia.org/wiki/File:Animal_locomotion._Plate_161_(Boston_Public_Library).jpg



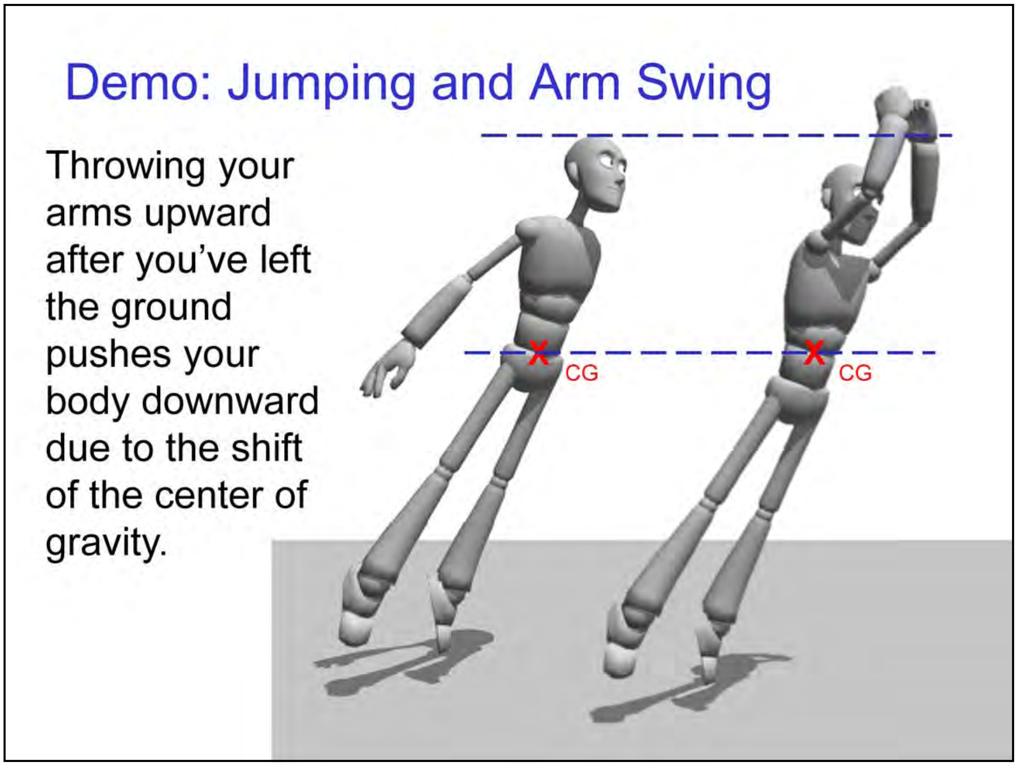
To demonstrate how the arm swing should be properly timed try jump normally, that is, swing your arms upward while feet are still on the ground.

Image by Corey Tom



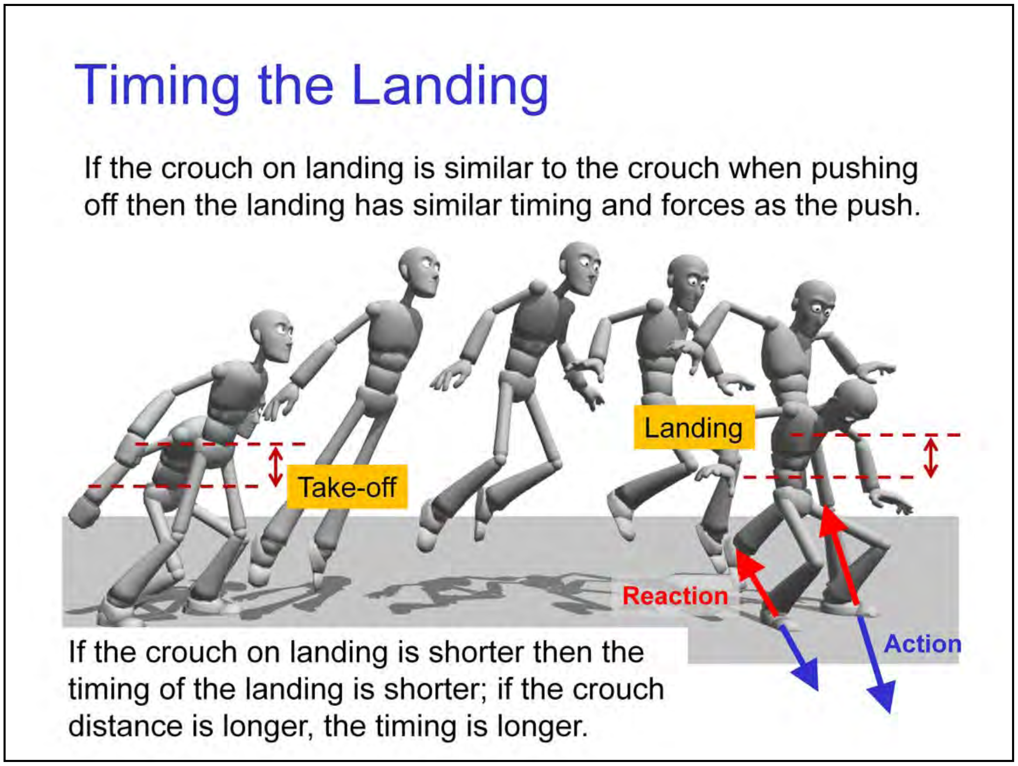
Now try swinging your arms upward after you leave the ground; you'll notice a big difference. It feels as if your whole body is pushed downward as you swing your arms upward.

Image by Corey Tom



Your torso moves downward as a reaction recoil to the force of throwing your arms upward. Another way to think of this is that raising the arms shifts the center of gravity higher in the torso. Be sure to try this demo yourself; once you experience the effect of doing it wrong you'll never forget to match the arm swing with the leg push when animating a jump.

Image by Corey Tom



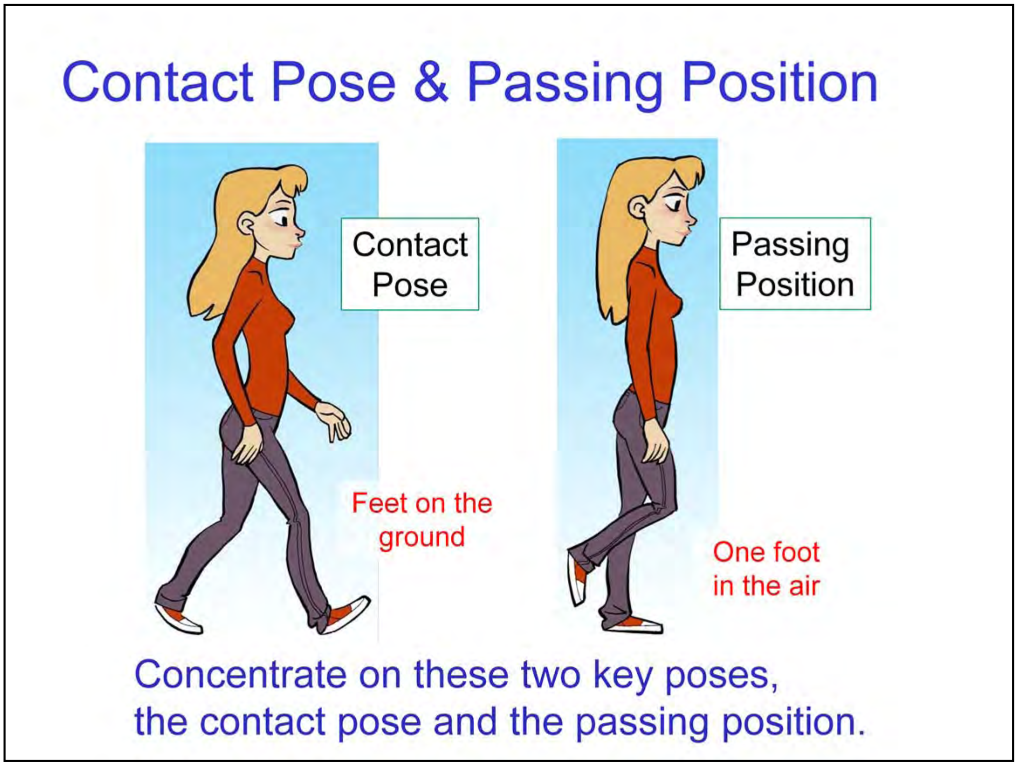
Finally, the timing of the settle, when the character lands, is related to the timing of the push. The two have similar timing if the distance that the character crouches on landing is similar to the distance it crouched when pushing off. If the crouch is deeper on the landing then the timing of the landing is slower. On the other hand, if the character lands with straight, stiff legs then the timing is quicker and the impact force is jarring (see the principle of Momentum and Force).

Image by Corey Tom

Part II: Applying the Principles

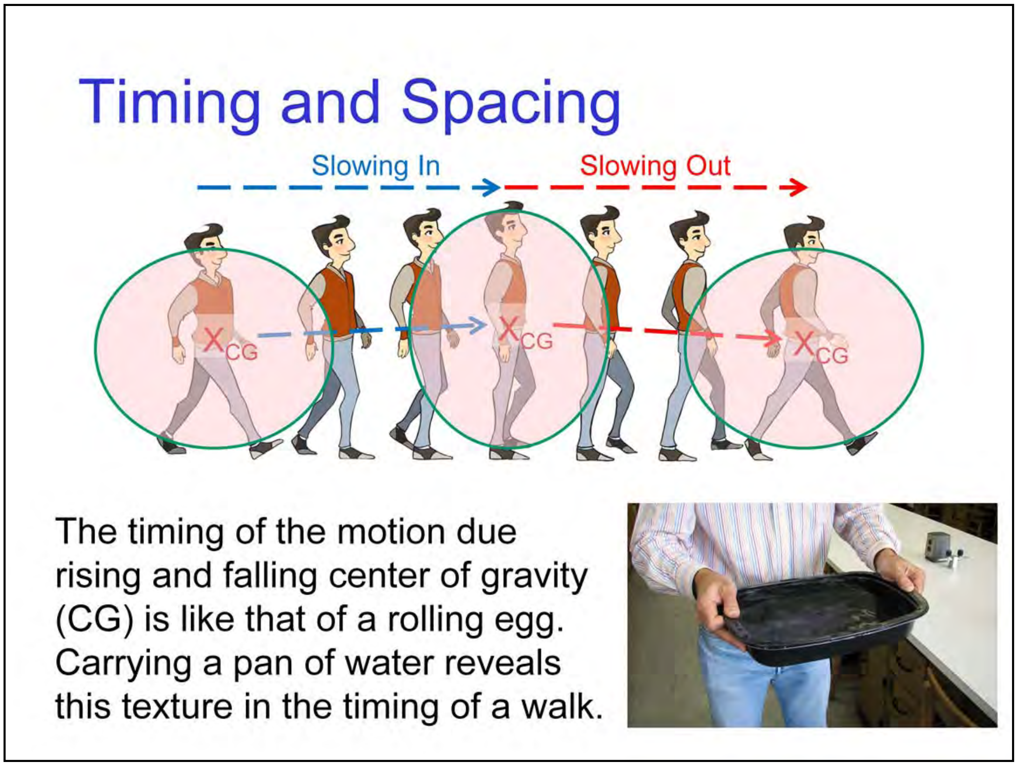
Walks

Finally, we look at how the Principles of Animation Physics apply to the animation of basic walk cycles.



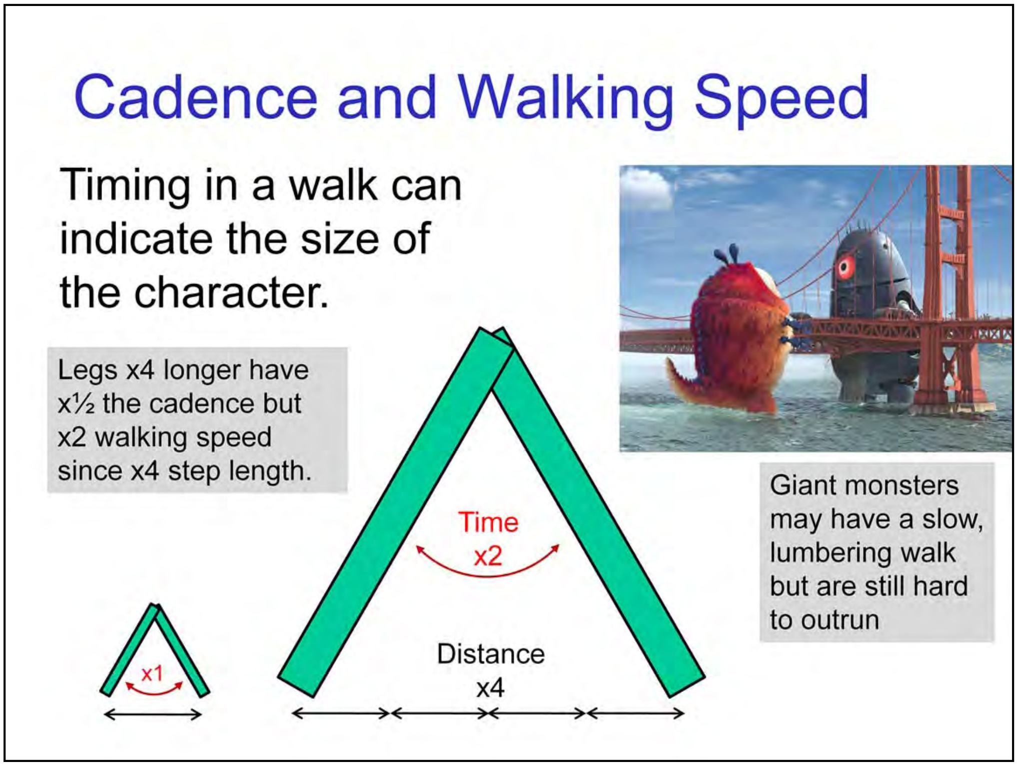
Walking is a cycle of steps alternating between the contact pose and the passing position. The bio-mechanics of walks, even just bipeds, could easily be a full three hour course by itself. Rather than trying to cover everything about character walks, we'll focus on examples that illustrate our six principles.

Illustration by Charlene Fleming

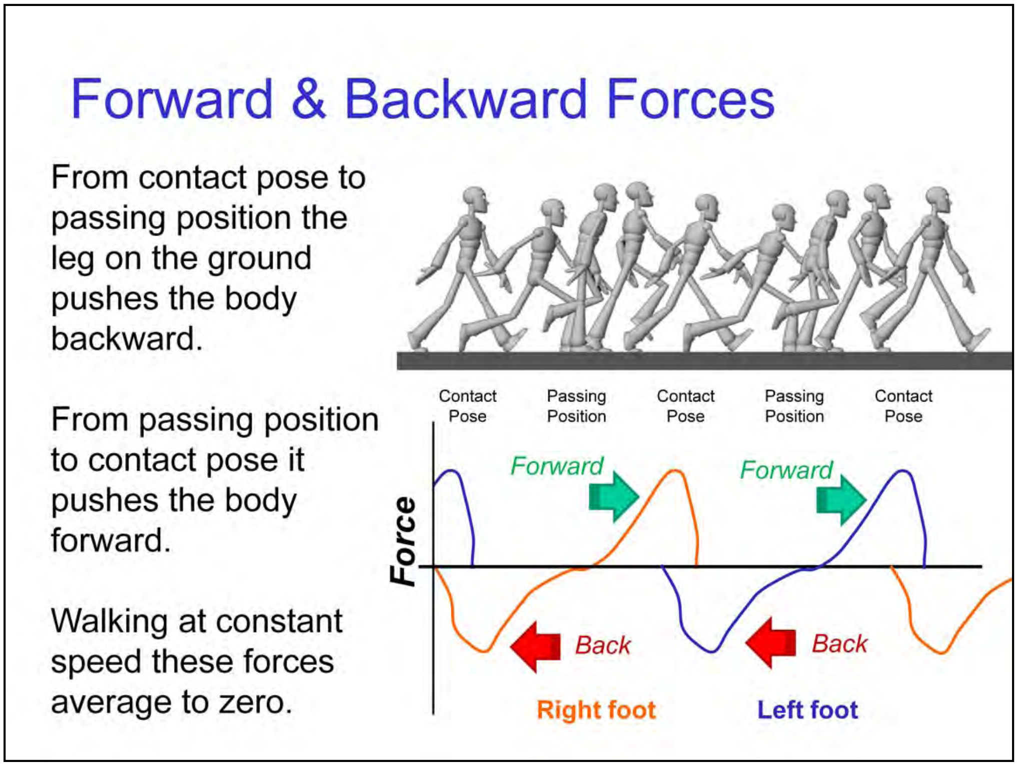


Although there is great variation in walks, the center of gravity generally rises as the character transitions into the passing position and it falls as the character leaves it. This causes the forward motion to slow a bit approaching the passing position and speed up again on leaving the passing position. The texture in this timing is subtle but the variation in speed is easily demonstrated by carrying a shallow pan of water while walking.

Illustration by Stephanie Lew

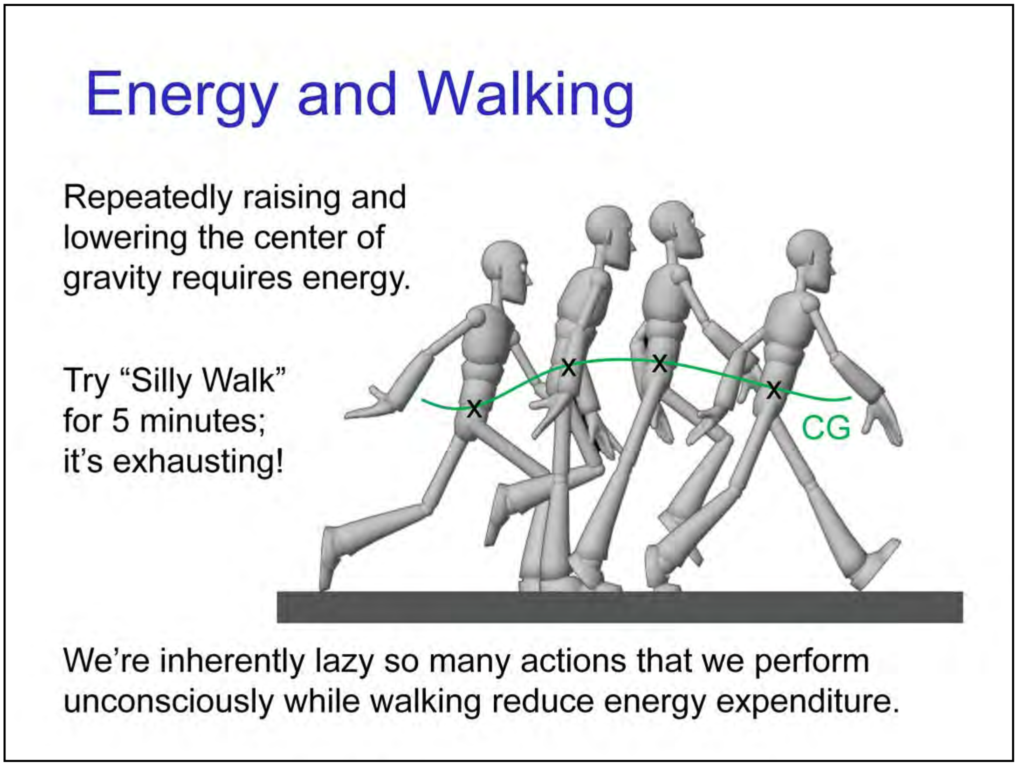


Long legs swing more slowly than short legs but also have a much longer step length. This is because the swinging of a pendulum follows the same scale relation as simple falling so we can apply the “Fourth Down at Half Time” rule. The short leg that’s a fourth the size takes half the time to swing back and forth.

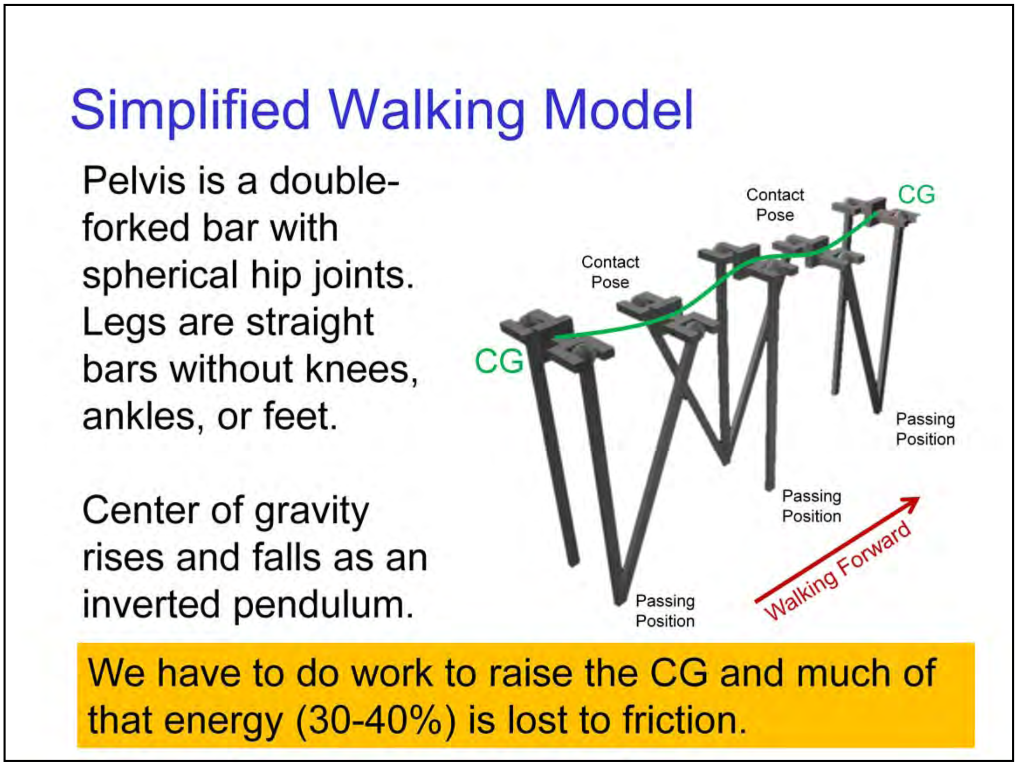


As with jumping, the character exerts action forces on the ground and relies on the resulting reaction forces to walk. From the contact pose to the passing position the force on the character is in the backward direction, slowing the motion. The necessity of this backward force is demonstrated by having the character step on a banana peel. From the passing position to the contact pose the force is in the forward direction, as expected. Interestingly, over the course of a full cycle the forces in the backward direction equal the forces in the forward direction. We know this because the speed is the same at the start and end of the cycle, so there's no change in momentum, so the total force is zero.

Illustration by Corey Tom

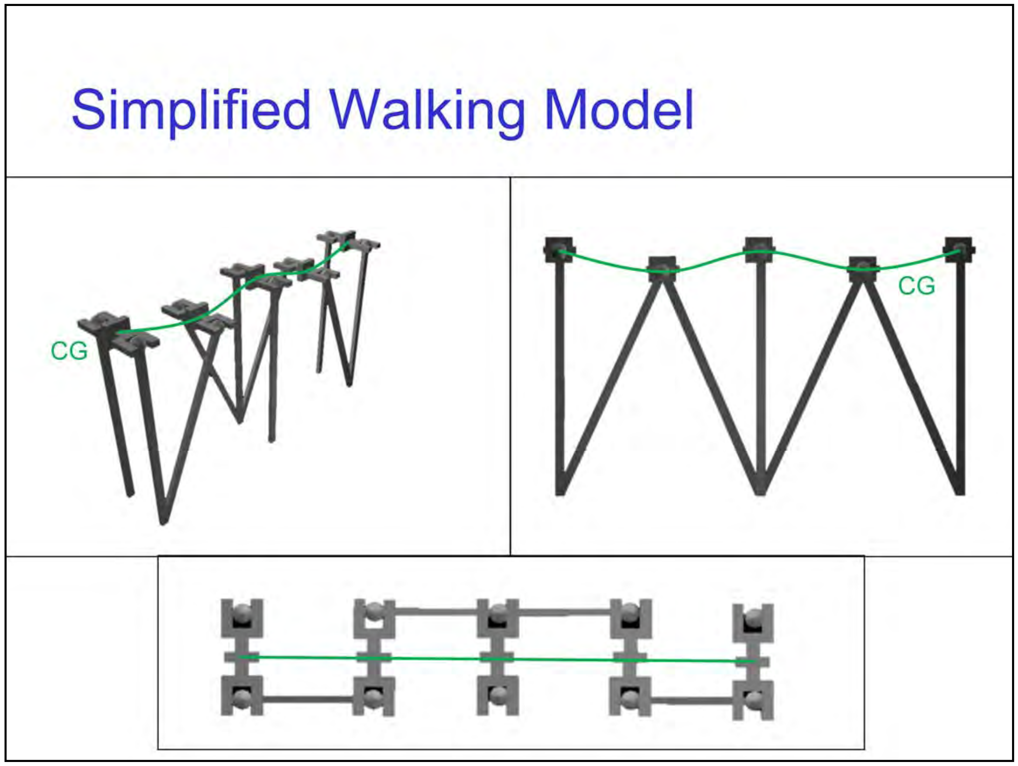


Raising the center of gravity while walking requires effort. This is best appreciated by walking for five minutes as they do in Monty Python's ``The Ministry of Silly Walks'' sketch; silly walking is surprisingly aerobic exercise. There are various subtle movements that the body performs to minimize the work required while keeping a constant step length and cadence.

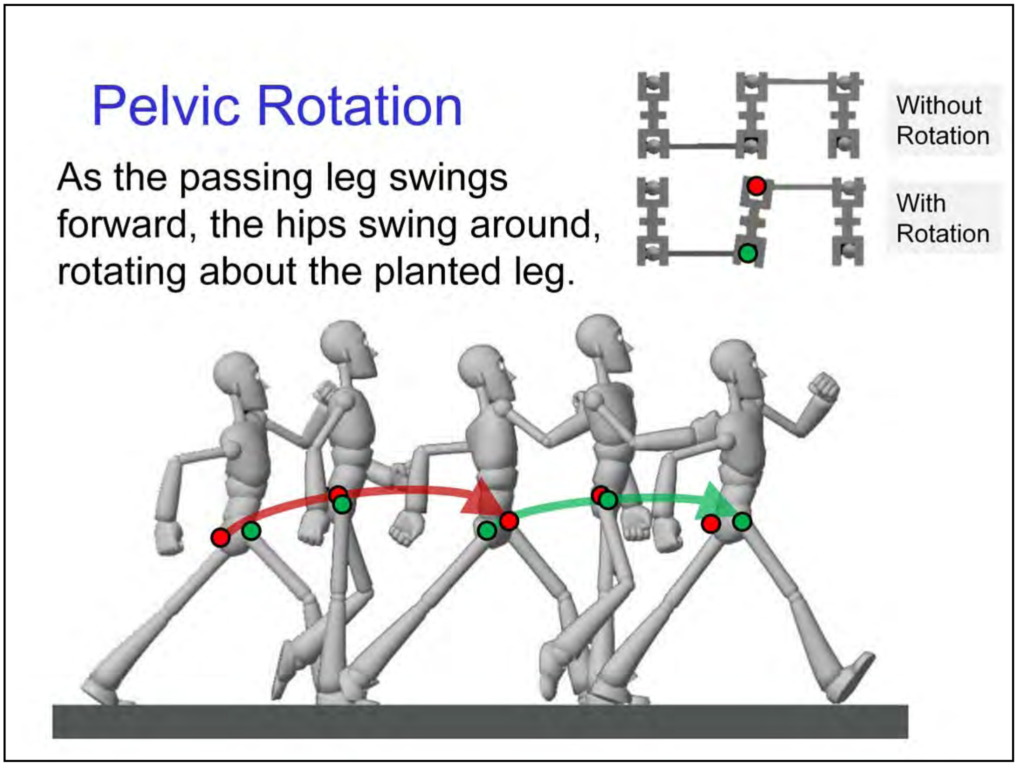


To analyze the mechanics of walking we'll use a simplified model for the hips and legs. In the simplest version of this model the pelvis is a double forked bar with spherical ball-and-socket hip joints; the legs are straight bars without knees, ankles, or feet. Walking with such legs is inefficient since the center of gravity falls entering the contact pose and rises again during the passing position. It takes significant effort (energy) by the character to repeat this up-and-down motion during the walk cycle.

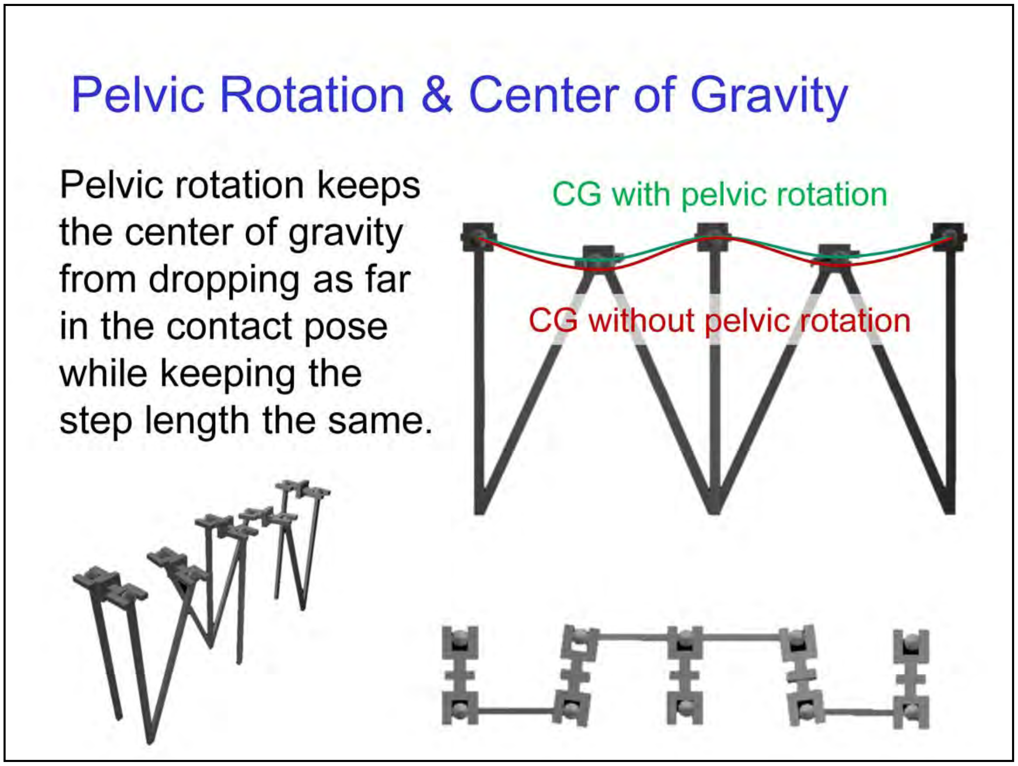
This walking model is discussed at length in "The Major Determinants in Normal and Pathological Gaits" by J. B. dec. M. Saunders, Verne T. Inman and Howard D. Eberhart, *J Bone Joint Surg Am.* 1953; 35:543-558



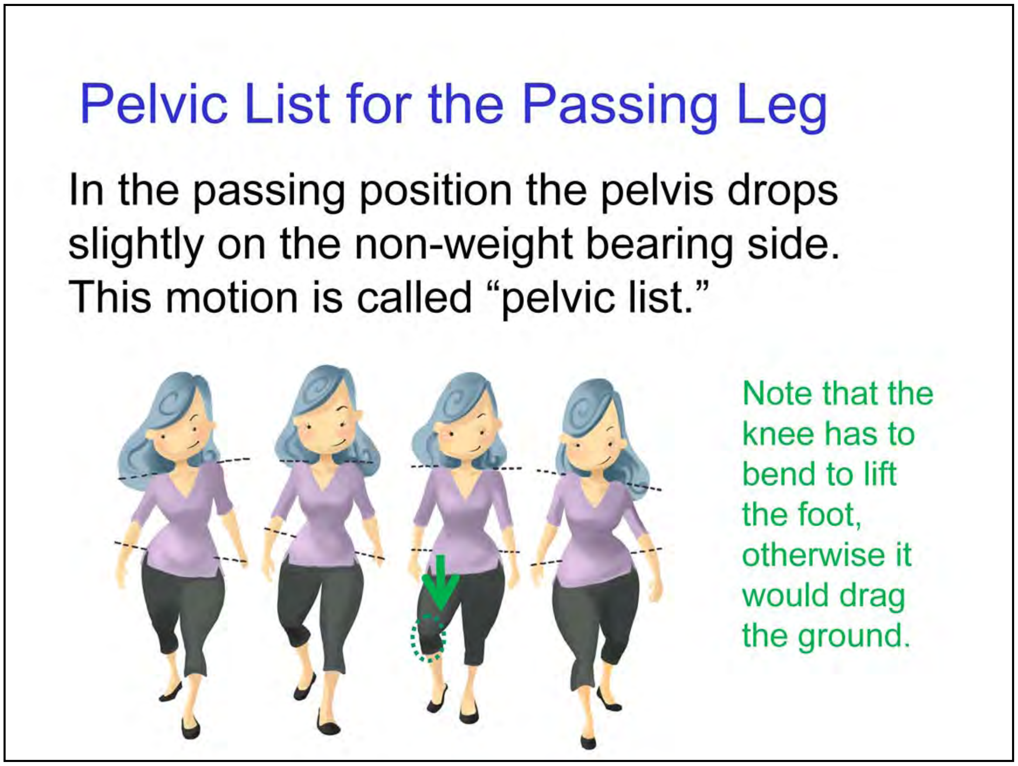
Simplified walking model in three-quarter, side, and top views.



To save energy the pelvis rotates to bring the forward leg out during the contact pose. This keeps the center of gravity from dipping as far down during the contact pose as compared to walking without a pelvic rotation. To allow this pelvic rotation to occur with minimal effort by the legs, the shoulders simultaneously rotate in the opposite direction.

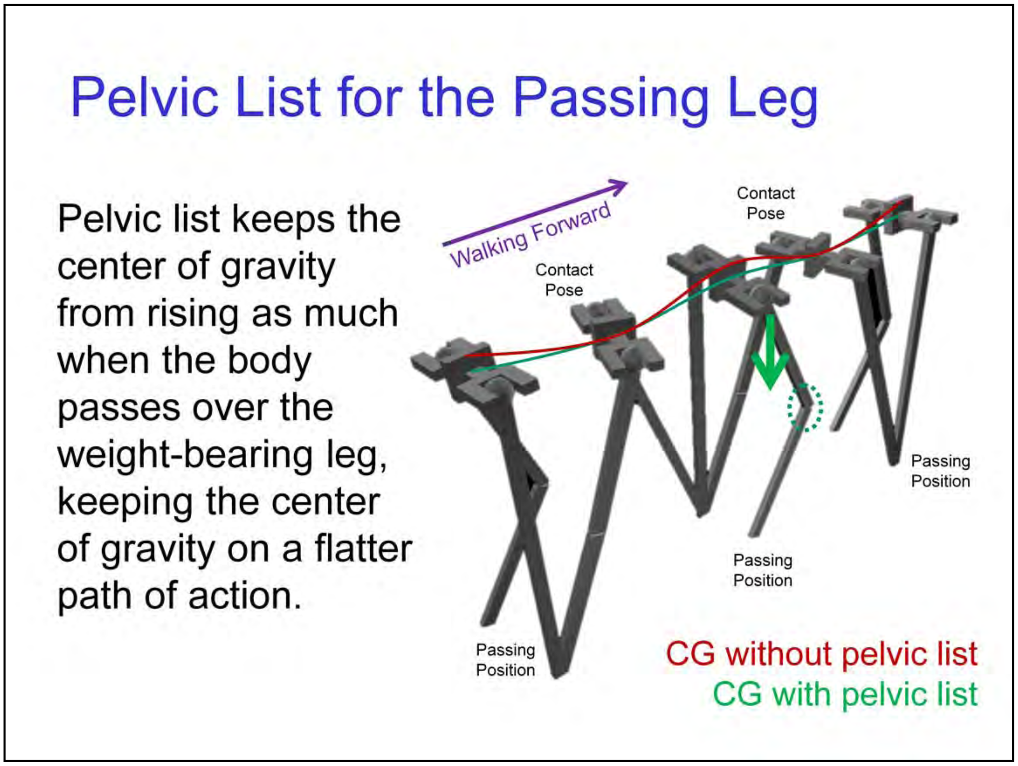


The pelvic rotation keeps the center of gravity from dropping as far in the contact pose by essentially increasing the effective length of the leg.

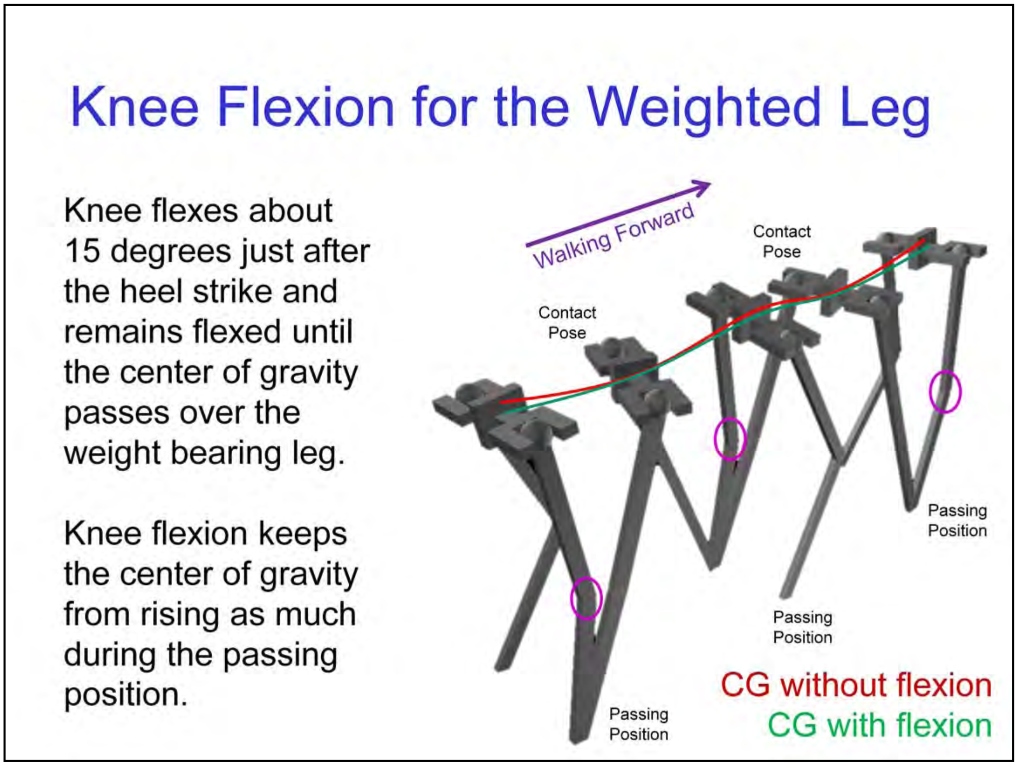


To keep the center of gravity from rising very much during the passing position the pelvis tilts to the side of the moving leg; this is called the “pelvic list.” Notice that the knee of the passing leg bends slightly to keep the foot from dragging the ground.

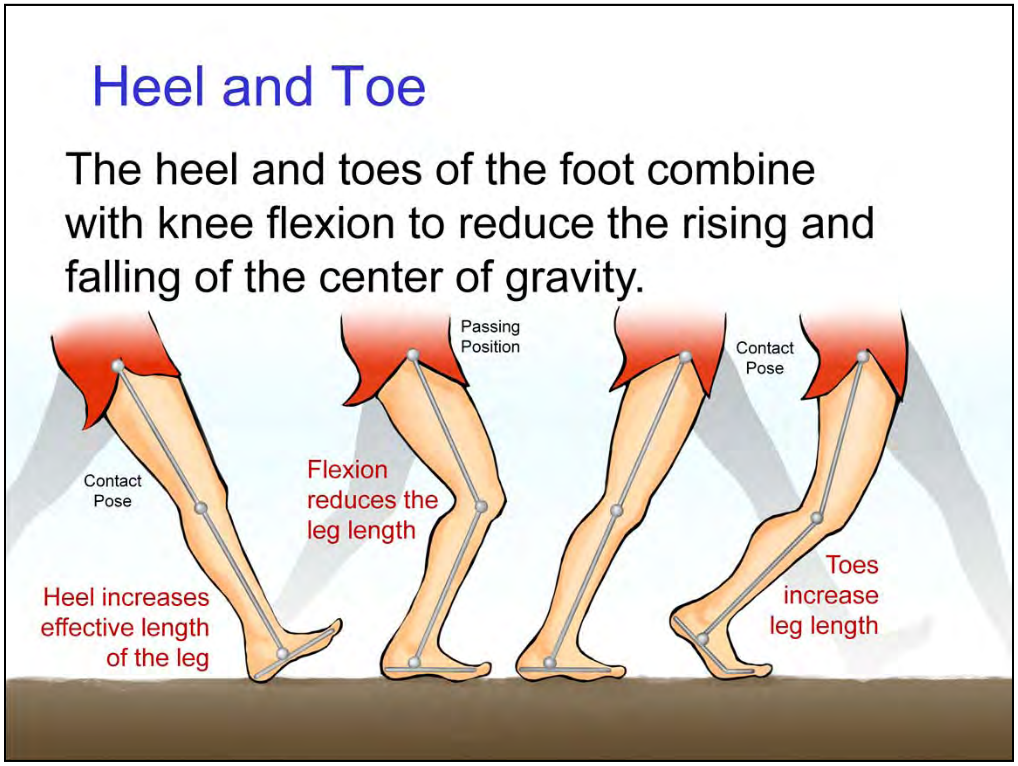
Illustration by Stephanie Lew



The path of action of the center of gravity is straighter due to pelvic list, which makes walking more energy efficient.



The knee of the weighted leg bends about 15 degrees just after the heel strike and remains flexed until after the passing position. Not only does this knee flexion make walking more efficient by reducing the rise of the center of gravity in the passing position, the force of impact on the body at heel strike is softened by this “squash.”

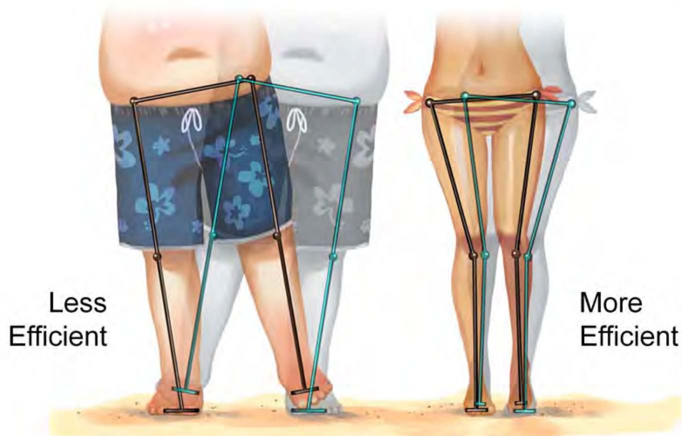


The heel and toes work together with the knee to adjust the effective length of the leg, lengthening it in the contact pose and shortening it in the passing position. This variation in length reduces the rising and falling of the center of gravity.

Illustration by Stephanie Lew

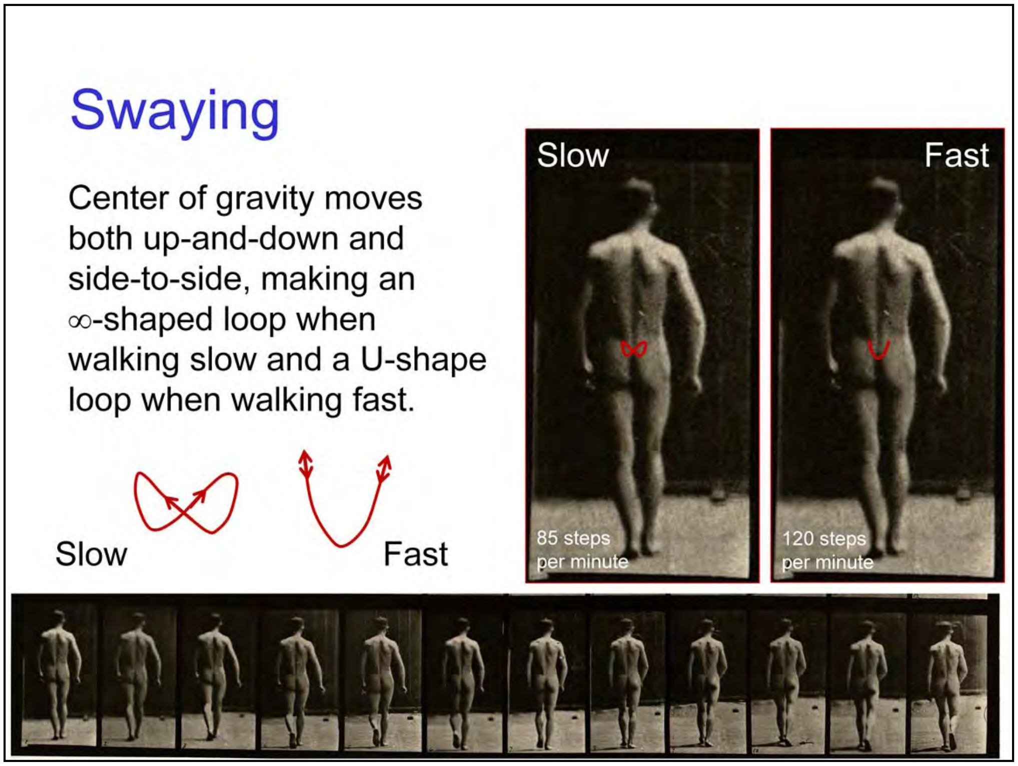
Stride Width

Shifting the center of gravity from left to right also requires work so a narrow stride is more efficient.



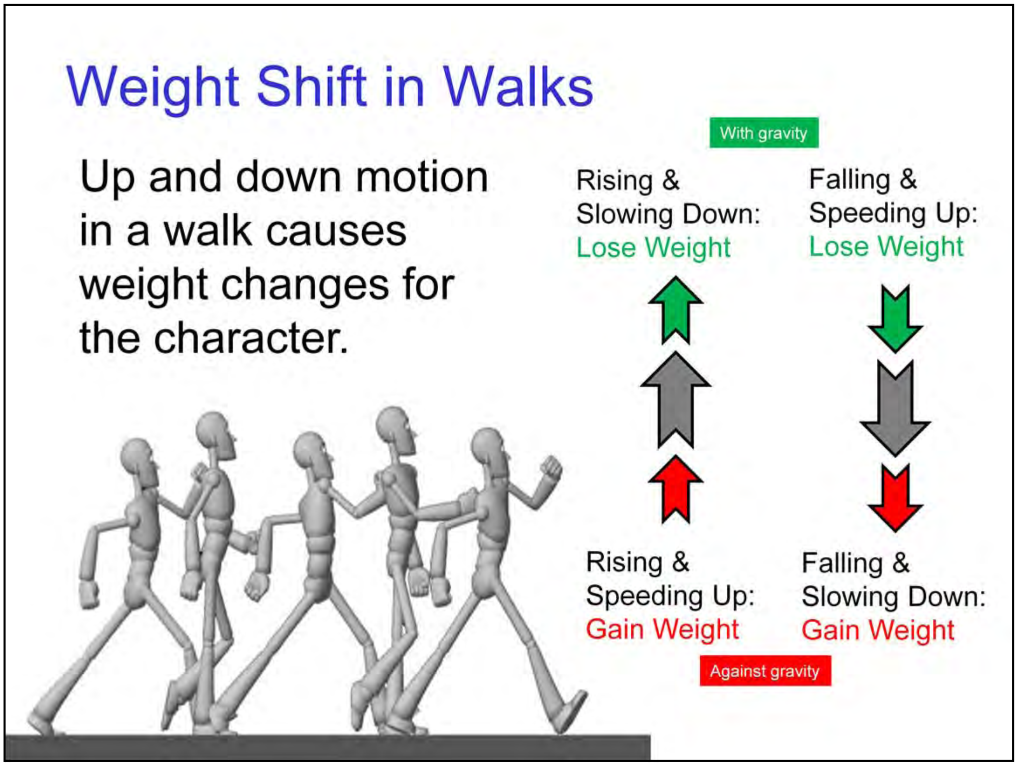
Besides the up-and-down motion of the center of gravity there is also some side-to-side motion. Walking is most efficient when this motion is minimized since forces have to be exerted by the legs to maintain it.

Illustration by Stephanie Lew



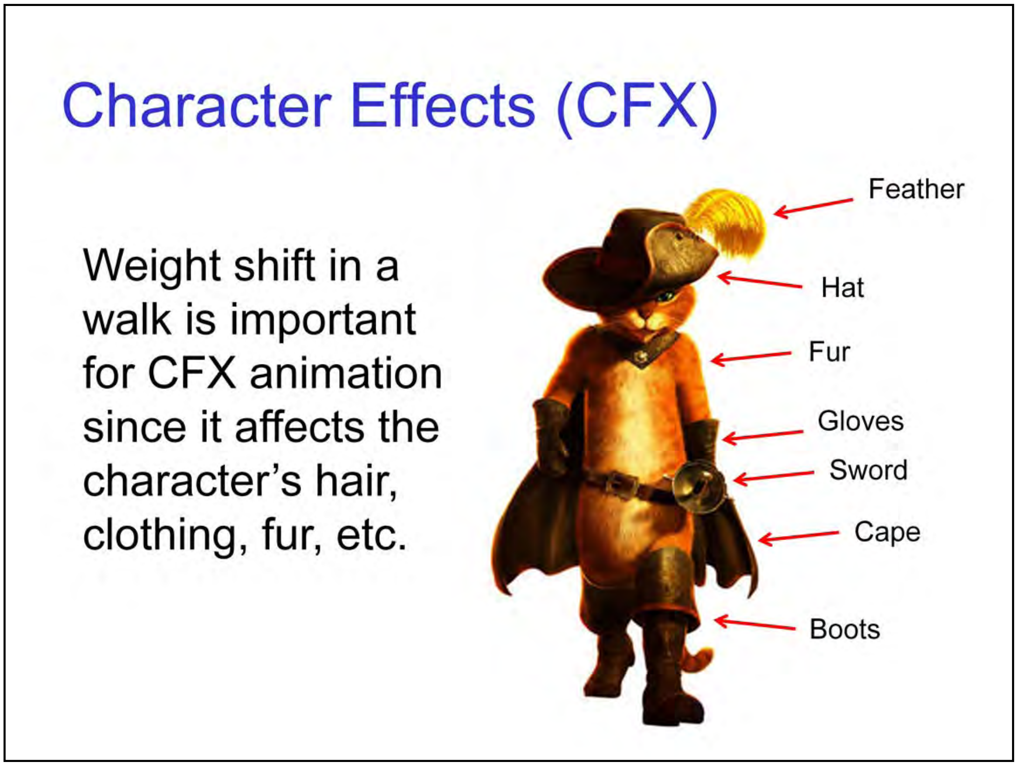
In walking there is always some side-to-side motion and it is synchronized with the up-and-down motion. For slow walks the center of gravity traces out an ∞-shaped pattern, as seen when the character is walking away from the camera. For fast walks the synchronization changes and the center of gravity swings almost like a pendulum, tracing a U-shaped pattern. Notice that this means a fast walk cannot be made into a slow walk by simply adding a few frames; there are subtle differences in the poses between the two types of walks.

Photo:
http://commons.wikimedia.org/wiki/File:Animal_locomotion._Plate_1_(Boston_Public_Library).jpg

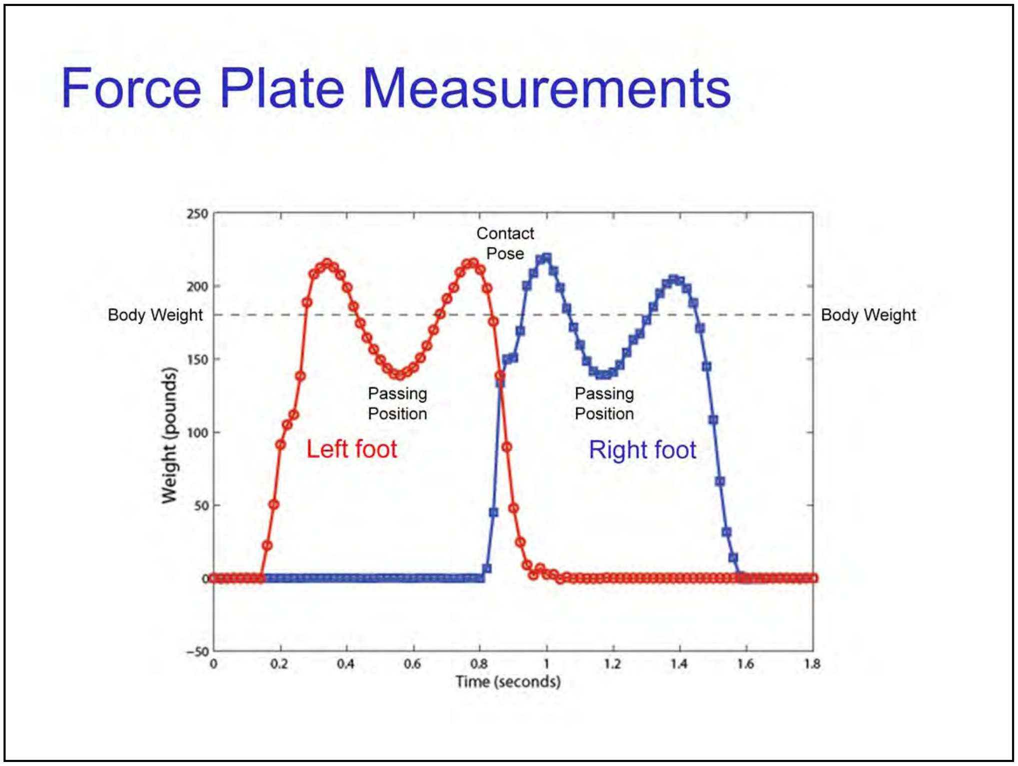


Up and down motion in a walk causes weight changes for the character (see the principle of Weight Gain and Loss).

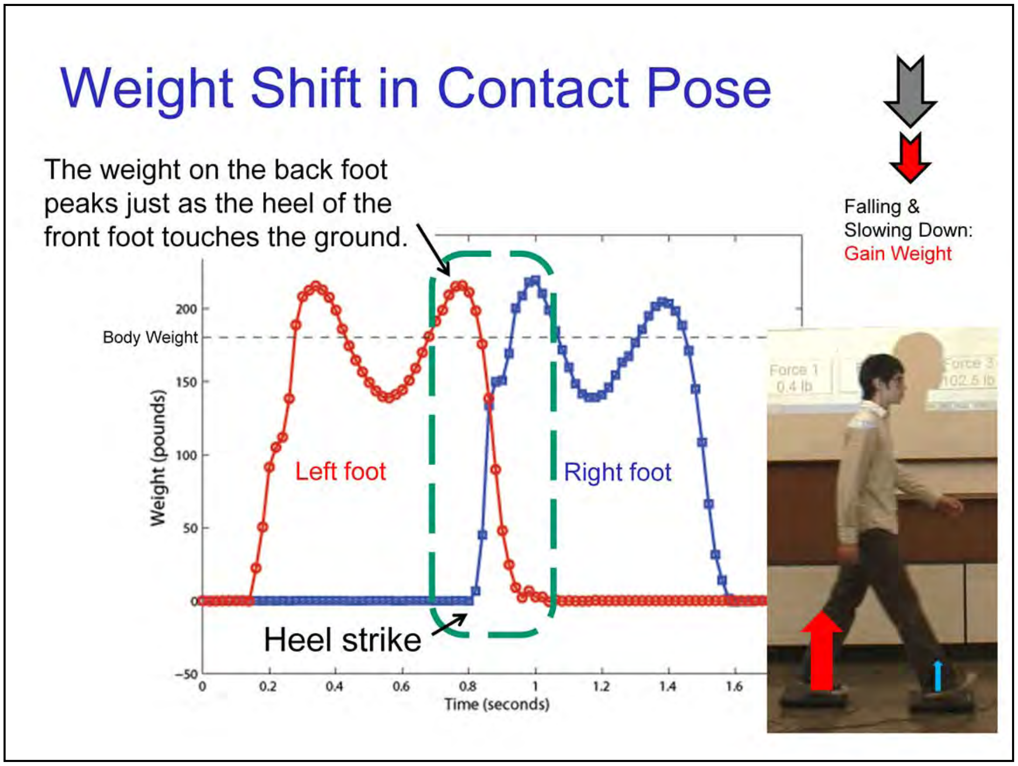
Image by Corey Tom



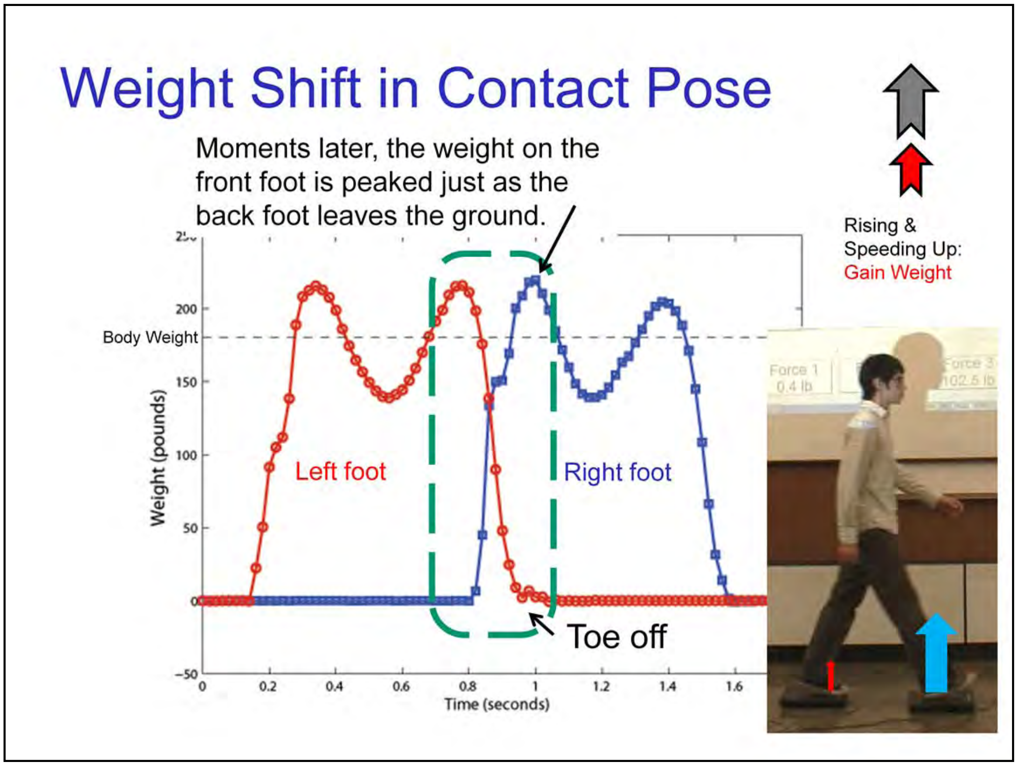
The weight variations that occur during a walk cycle are especially noticeable in the overlapping action of the character's hair, clothes, flesh, etc.



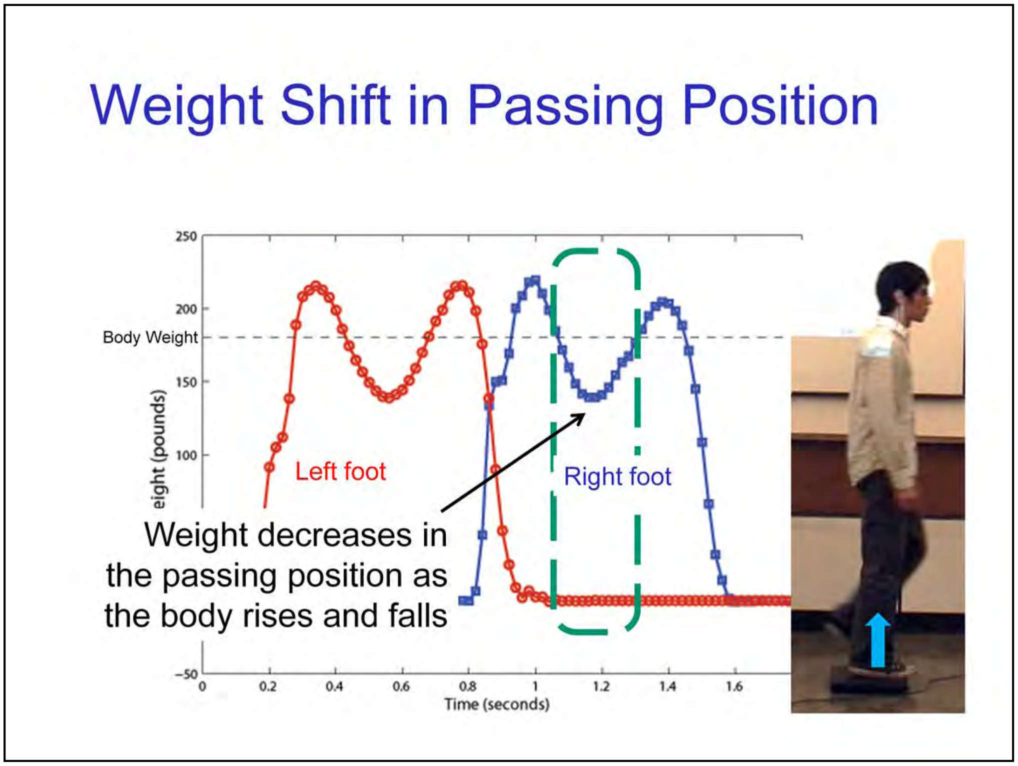
The graph shows the weight shift from one foot to the other while walking. Notice that due an up-and-down motion of the center of gravity there is significant weight gain and weight loss during the cycle. This graph is for an average walk (about one second per cycle). For slower walks the weight variations are smaller; for very slow walks the weight on the planted foot is nearly constant from heel strike to toe lift. For brisk walks the weight can vary dramatically, dipping to almost zero during the passing position and spiking to over twice the normal weight around the contact pose.



Let's look at the weight shift during the contact pose. At the moment that that the heel strike occurs with the forward foot there is a rise in the weight on the back foot. In the middle of the contact pose the combined weight for the two feed is significantly greater than the body's resting weight. This weight gain occurs since the body is first falling and slowing down, entering the contact pose, then rising and speeding up, leaving the contact pose (see the principle of Weight Gain and Loss).



At the end of the contact pose the back foot leaves the ground and the weight shifts entirely to the front foot. At this point the weight on that foot is greater than the body's resting weight but starts to drop as we transition into the passing position.



The dips in the weight occur during the passing position as the character slows while rising then speeds while falling (see the principle of Weight Gain and Loss).

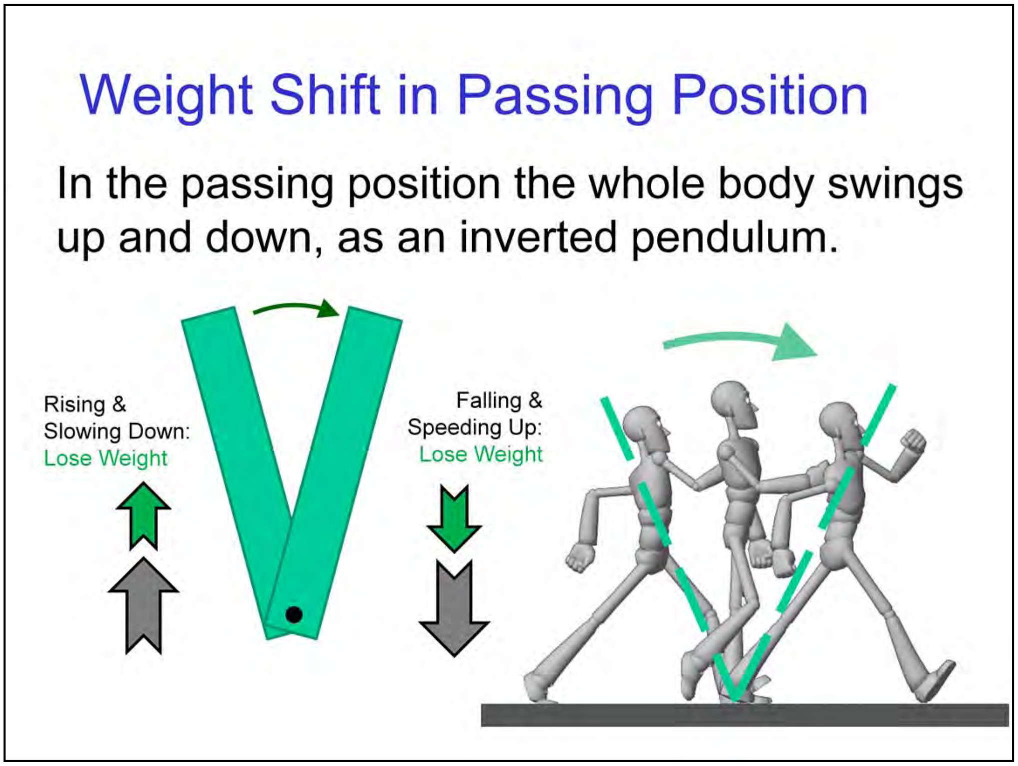


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Image by Corey Tom

Concluding Remarks

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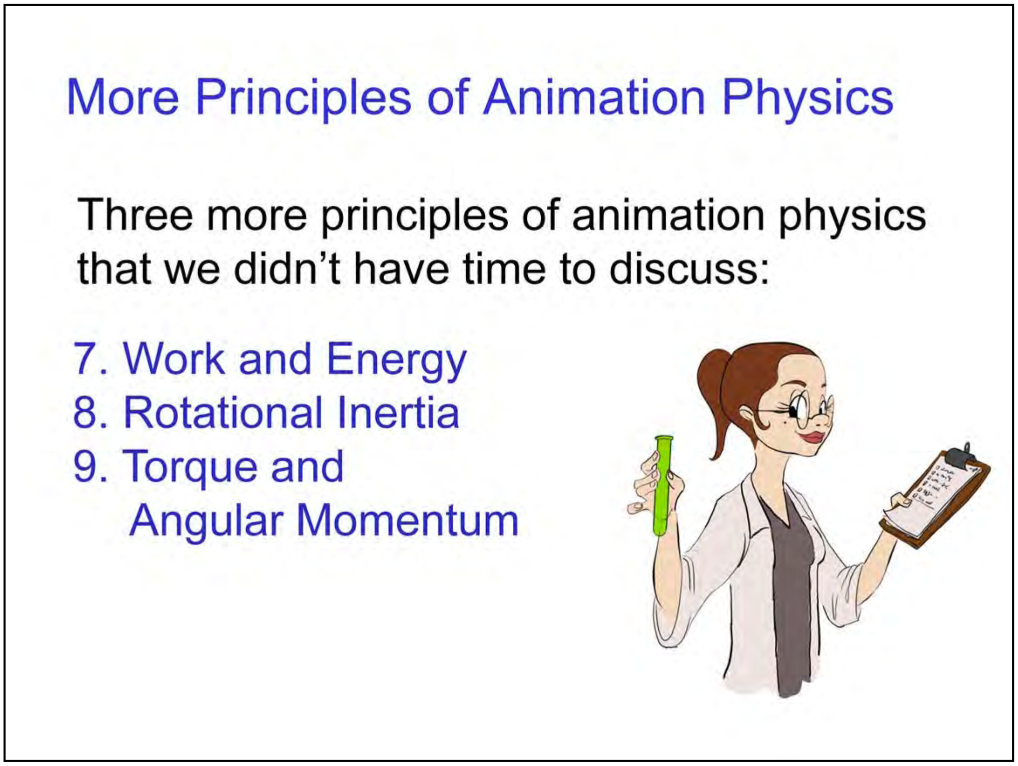


While this course covers what I believe to be the most important principles of animation physics, if we had more time then we would add three more to our list. Specifically,

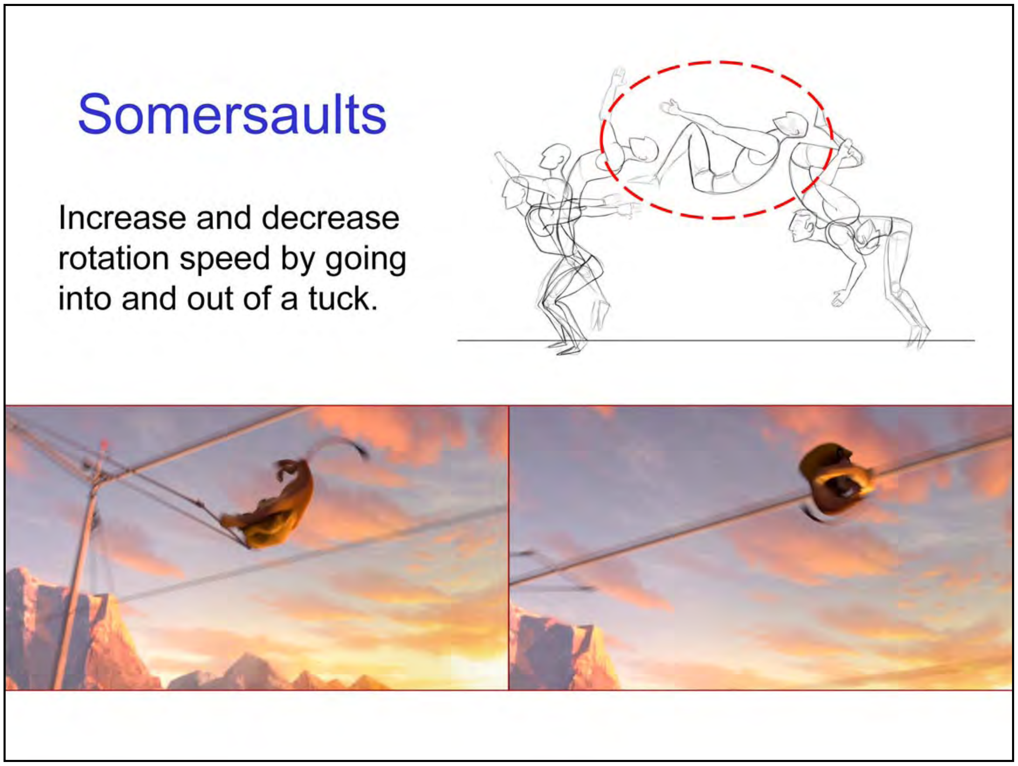
- Work and Energy
- Rotational Inertia
- Angular Momentum and Torque

We already touched on these briefly in the discussion on walks. For example, it takes work to raise the center of gravity and torque to rotate the hips and shoulders. And understanding rotational inertia is essential for more complicated character rotations, such as a back somersault or a cat twisting to land on its feet.

Illustration by Rebbaz Royee

Somersaults

Gymnasts can controls their rotation by changing their pose. For example, in going from a layout into a tuck position the rotation can be five times faster.



Gymnasts can controls their rotation by changing their pose. For example, in going from a layout into a tuck position the rotation can be five times faster.

Illustration by Dora Roychoudhury

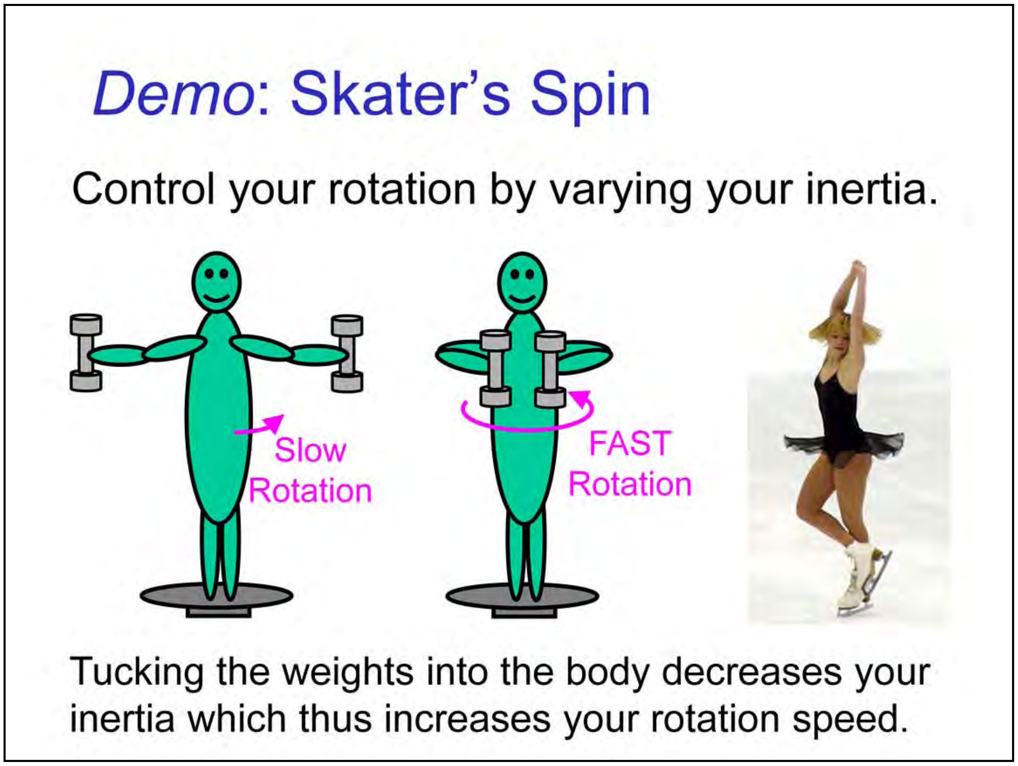


Figure skaters can increase (or decrease) their spin by bringing their arms and legs close to (or far from) their torso, which decreases (or increases) their rotational inertia. This is a classic physics demonstration done on a rotating platform. Notice that in the photo of the ice skater we can tell that she's rapidly spinning (or has just stopped spinning) because the centrifugal effect seen in her hair and skirt.

Skater photo: http://commons.wikimedia.org/wiki/File:Elena_Sokolova_04_NHK_2.jpg

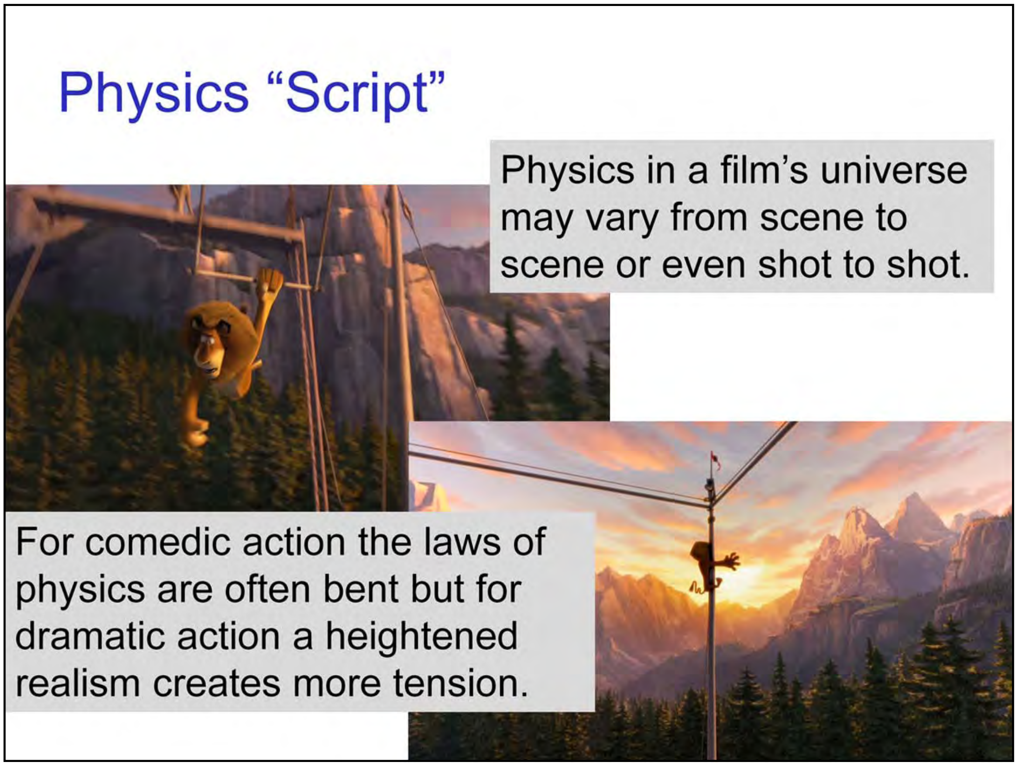
Phys-Dev for Animation

Each film has a physical design (phys-dev) for the film’s world and the characters in it.

For example, Master Mantis can violate the Action / Reaction principle when he fights.

The principles of animation physics are useful at all stages, including story, layout, and character design. In the early stages of production the director works with a variety of artists on the visual design (vis-dev). Besides the general look of the film there are many decisions regarding the universe in which the story takes place. For example, animals talk in *Kung Fu Panda* but they don't in *How to Train Your Dragon*.

Similarly, there is also the physical design (phys-dev) of the film's universe. Take, for example, the character of Master Mantis in *Kung Fu Panda*. In order for Mantis to fight villains who are a hundred times his size the principle of action-reaction is suspended. This decision leads to other phys-dev questions: Does action-reaction apply to the other heros? What about the villains or ordinary citizens? Are there scenes where Mantis will obey action-reaction?



One can also think of a ``physics script'', similar to a color script, that indicates the variation in the physics in each scene. For comedic action the laws of physics are often bent but for dramatic action a heightened realism creates more tension. Many of these phys-dev decisions are made instinctively by the director and when the design is successful the film's universe feels natural, immersing the audience in the story. So use the principles wisely and may the forces be with you.

Images and videos from *Madgascar 3: Europe's Most Wanted*, *Kung Fu Panda*, and *Monsters versus Aliens*, courtesy of Dreamworks Animation

Unless otherwise noted, photos are by the author.