

## Program 12 / Chapter 14

# Doppler effect for a moving observer

When a wave source and receiver are moving relative to each other, the frequency of the waves arriving at the receiver is changed. This phenomena, known as the Doppler effect, accounts for the difference heard when a siren or whistle is moving towards you or away from you. The program in this section computes the wave displacement measured by a moving observer, illustrating the change in frequency.

Theory When either the source or receiver of waves is moving, the frequency of the waves observed at the receiver is shifted; this phenomenon is called the Doppler effect. In this section we specifically consider the scenario where the wave source (e.g., a loud speaker) is stationary while the observer (e.g., a listener) is moving either directly towards or away from the source (this is Case B in section 14–7).

Your textbook gives the frequency measured by the moving observer to be

$$f = \begin{cases} f_0 \left(1 + \frac{|v_r|}{v}\right) & \text{if moving toward source} \\ f_0 \left(1 - \frac{|v_r|}{v}\right) & \text{if moving away from source} \end{cases}$$

where  $f_0$  is the frequency of the waves at the source (i.e., frequency measured by a stationary observer),  $v_r$  is the velocity of the observer and  $v$  is the wave speed. For example, if the observer is moving away from the source with a speed of  $v_r = v/2$ , then the frequency measured by the moving observer is half the source frequency.

We will consider a plane wave where the wave displacement as a function of position and time is given by,

$$z(x, t) = \sin(kx - \omega t)$$

where  $k$  is the wavenumber and  $\omega$  is the angular frequency ( $\omega = 2\pi f_0$ ). The wave displacement measured by a moving observer is

$$z_r(t) = \sin(kx_r(t) - \omega t)$$

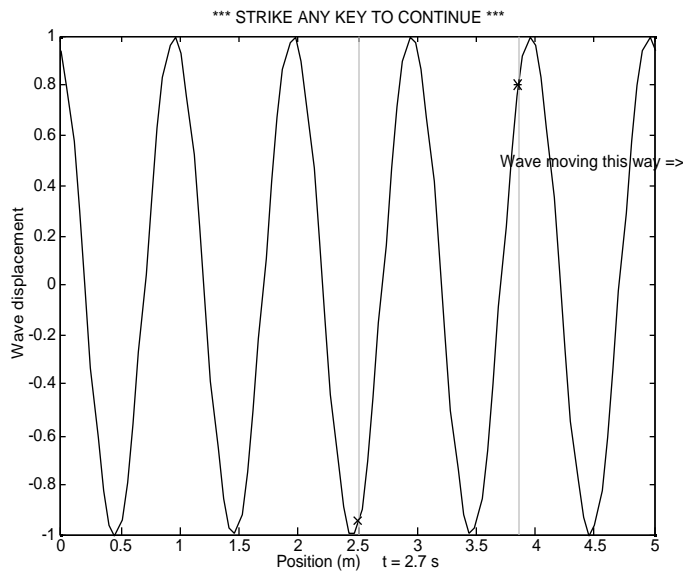
where  $x_r$  is the position of the observer. The program in this section measures and plots the wave displacement measured by both a moving and a stationary observer, illustrating the Doppler effect.

Program The MATLAB program `doppler`, which illustrates the Doppler effect for a moving observer, is outlined below:

- Initialize variables (e.g., speeds of the wave and moving observer).
- Set up the graphics.
- Loop over the number of steps, periodically plotting a snapshot.
  - Compute the wave displacement at location of the moving observer.
  - Compute the wave displacement at location of the stationary observer.
  - If it is time to plot a snapshot then,

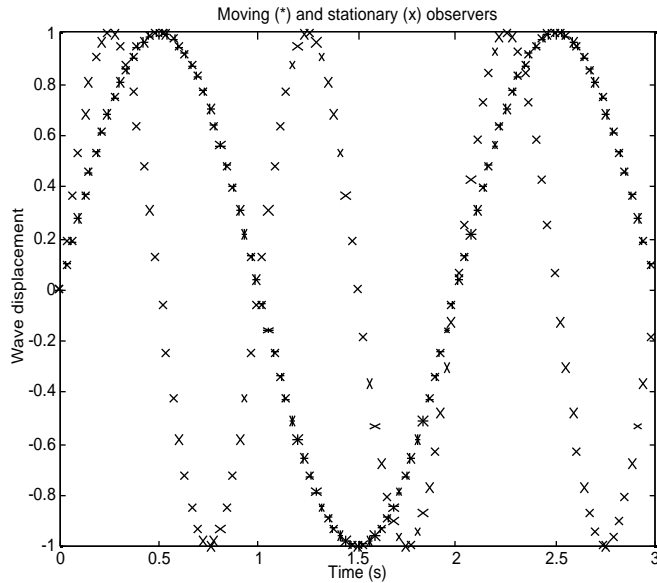
- Compute the wave displacement versus  $x$  for the current time.
- Plot the wave amplitude versus position.
- Mark the location of each observer with a vertical bar.
- Mark the wave amplitude at the moving and stationary observers.
- Record the time and wave amplitude at each observer.
- Increment the time and the position of the moving observer.
- Plot wave displacement versus time as seen by each observer.

For an observer speed of 0.5 m/s (the wave speed is 1 m/s), one of the snapshots produced by the program is shown below. The vertical bars indicate the positions of the observers; the stationary observer is located at the center of the graph while the moving observer is near the right edge.



The wave moves from left to right so the moving observer is moving away from the source. The wave amplitude is marked by an  $\times$  for the stationary observer and by an asterisk for the moving observer.

After all the snapshots have been computed, the program pauses and prompts you to strike any key to continue. After proceeding, the graph of the wave amplitude at each observer as a function of time is produced.



In this case, since the moving observer is moving away from the wave source at half the wave speed, that observer measures a frequency that is half the frequency measured by the stationary observer. This result is found in the above plot since the stationary observer records 3 full oscillations while the moving observer records  $1\frac{1}{2}$  oscillations.

### Exercises

1. (I) Run the `doppler` program for the following observer speeds and print out the final plot (wave displacement versus time) for each case: (a)  $v_r = -0.9$  m/s; (b)  $v_r = -0.5$  m/s; (c)  $v_r = 0.0$  m/s; (d)  $v_r = 0.25$  m/s; (e)  $v_r = 0.5$  m/s; (f)  $v_r = 0.9$  m/s.
2. (I) Increase the horizontal scale used for plotting (the variable `xMax`) and run `doppler` program for the following observer speeds; print out the final plot (wave displacement versus time) for each case: (a)  $v_r = -2.0$  m/s; (b)  $v_r = -1.5$  m/s; (c)  $v_r = 1.5$  m/s; (d)  $v_r = 2.0$  m/s.
3. (II) Modify the `doppler` program so that it plots the wave displacement versus time as predicted by the Doppler effect, i.e., plot the function

$$z(t) = \sin(2\pi f t)$$

Run your program and plot the cases listed in exercise 1.

4. (II) Modify the `doppler` program so that the source, instead of producing a sine wave, produces the step function,

$$z(x, t) = \begin{cases} +1 & \text{if } (\sin(kx - \omega t) > 0) \\ -1 & \text{if } (\sin(kx - \omega t) < 0) \end{cases}$$

Run your program and plot the cases listed in exercise 1.

5. (III) Modify the `doppler` program so that it simulates a moving source with a stationary observer. Run your program and print out the wave displacement versus time graph for the following source speeds:  $v_s = -0.9$  m/s,  $-0.5$  m/s,  $0.0$  m/s,  $0.25$  m/s,  $0.5$  m/s and  $0.9$  m/s.

6. (III) Modify the `doppler` program so that it simulates a moving source and a moving observer. Run your program and print out the wave displacement versus time graph for the following source and receiver speeds:  $v_s = -0.5$  m/s  $v_r = -0.25$  m/s;  $v_s = -0.25$  m/s,  $v_r = 0.25$  m/s and  $v_s = 0.25$  m/s,  $v_r = -0.25$  m/s.

### Listing

`doppler.m`

```
% doppler - Compute doppler effect for moving observer
clear all; help doppler; % Clear memory; print header

%@ Initialize variables
kSource = 2*pi; % Wave number of the source (m^-1)
wSource = 2*pi; % Frequency (omega) of source (Hz)
cWave = wSource/kSource; % Speed of the wave
fprintf('Speed of the wave is %g m/s\n',cWave);
vObs = input('Enter observer speed (m/s): ');
t = 0; % Initial time
tIncrement = 0.03; % Time increment between steps

%@ Set up graphics
clf; figure(gcf); % Clear figure; Bring figure window forward
xMax = 5; % Maximum value of x for plots
NPoints = 100; % Number of points to plot when drawing the wave
for i=1:NPoints % Compute values of x for plot
    x(i) = (i-1)/(NPoints-1)*xMax;
end
xObs0 = xMax/2; % Initial location of observer (center of plot)
xObs = xObs0; % Location of observer

%@ Loop over the number of steps, periodically plotting a snapshot
NSteps = 100; % Number of steps
ShotInterval = 10; % Snapshot plotted once every 10 steps
for iStep=1:NSteps

    %@ Compute the wave displacement at location of moving observer
    zObs = sin(kSource*xObs - wSource*t);
    %@ Compute the wave displacement at location of stationary observer
    zStat = sin(kSource*xObs0 - wSource*t);

    %@ If it is time to plot a snapshot then,
    if( rem(iStep,ShotInterval) == 1 )
        %@ Compute and plot the wave displacement vs x for the current time
        z = sin(kSource*x - wSource*t);
        plot(x,z,'y-');
        axis([0, xMax, -1, 1]); % Set the axis limits
        xlabel(sprintf('Position (m) t = %g s',t)); % X-axis label
        ylabel('Wave displacement'); % Y-axis label
        title('Observers at positions marked with vertical lines');
        hold on;
        %@ Mark the location of each observer with a vertical bar
        plot([xObs xObs],[ -1 1], 'r:');
        plot([xObs0 xObs0],[ -1 1], 'b:');
```

```

    %@ Mark the wave amplitude at the moving and stationary observers
    plot(xObs,zObs,'r*');
    plot(xObs0,zStat,'bx');
    %@ Write text string on graph (indicating wave direction)
    text(1.0+cWave*t,0.5,'Wave moving this way =>');

    drawnow;    % Draw the graph
    hold off;
end

    %@ Record the time and wave amplitude at each observer
    tPlot(iStep) = t;    % Record time for final plot
    zObsPlot(iStep) = zObs;    % Record zObs for final plot
    zStatPlot(iStep) = zStat; % Record zStat for final plot

    %@ Increment the time and the position of the moving observer
    t = t + tIncrement;    % Update time
    xObs = xObs + vObs*tIncrement; % Update moving observer position

end

title('*** STRIKE ANY KEY TO CONTINUE ***');
pause;    % Pause before drawing the next plot

    %@ Plot wave displacement versus time as seen by each observer
    hold off;    % Turn off the plotting hold
    plot(tPlot,zObsPlot,'r*',tPlot,zStatPlot,'bx')
    xlabel('Time (s)');
    ylabel('Wave displacement');
    title('Moving (*) and stationary (x) observers');

```

