

Program 27 / Chapter 37

Spherical Mirrors

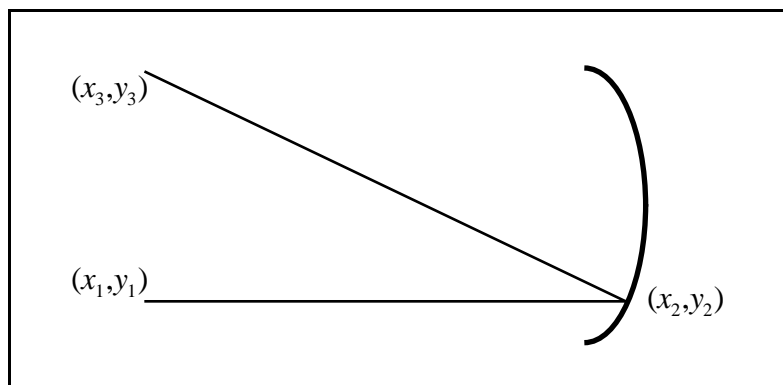
Optical instruments consist primarily of mirrors and lenses. In this section, we use a program to draw the rays reflected by a concave spherical mirror.

Theory A concave spherical mirror reflects paraxial rays towards a common point, called the focal point (see Section 37–2 and Figure 37–8 in your textbook). This focal point is located midway between the center of curvature and the mirror surface. Note that the focus for a spherical mirror is imperfect; the rays pass near but not exactly through the focal point. Only when the angle of reflection is small do the rays converge to a tight focus.

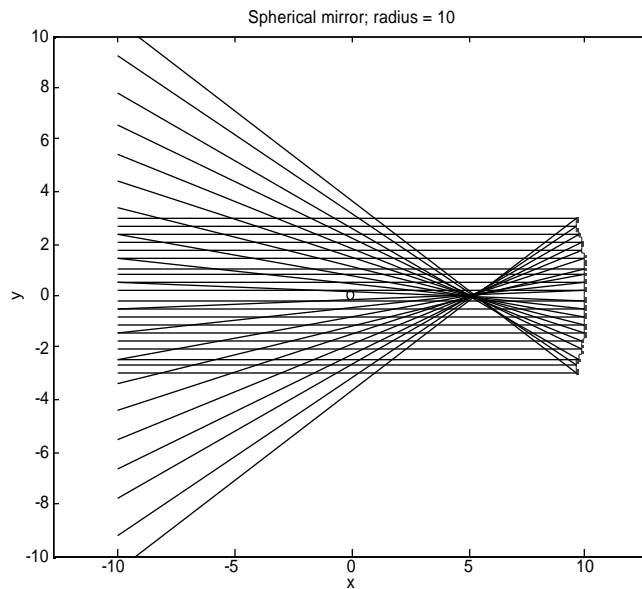
Program The MATLAB program `mirror`, which computes rays reflected by a concave spherical mirror, is outlined below:

- Initialize variables (e.g., mirror radius, number of rays).
- Set up for plotting rays.
- Draw the mirror surface.
- Loop over the rays to be drawn.
 - Find the ray's point of intersection with the mirror.
 - Compute incident and reflection angles (relative to x -axis).
 - Compute ray's final point back on vertical plane to the left.
 - Draw the ray using its three points.

The geometry used by the program is illustrated below. Each ray drawn by the program is composed of three points. The first point, located at (x_1, y_1) , is placed on the left side of the plot; rays travel horizontally to the right towards the mirror (paraxial rays). The second point is on the surface of the mirror at (x_2, y_2) . The angle of the mirror surface at this point is computed and the ray's new direction is found using the law of simple reflection (i.e., angle of incidence equals angle of reflection). Finally, given this new direction angle, the third point for the ray is computed.



For a maximum ray height (i.e., vertical distance from x axis) of 3.0, the `mirror` program produces the graph shown below.



Notice that the rays focus at a point located half way between the origin of the circle (located at the origin and marked with an O) and the mirror surface; compare with Figure 37–8 of your textbook.

Exercises

1. (I) Run the `mirror` program (using 20 rays and a mirror radius of 10.0) and print out the graph produced for the following values of maximum ray height: (a) 1.0; (b) 2.0; (c) 4.0; (d) 6.0; (e) 8.0 and (f) 10.0.
2. (I) Running the `mirror` program with a maximum ray height close to the mirror radius produces erroneous results (e.g., try a maximum ray height of 9.0). Explain the origin of the problem and suggest a possible correction.
3. (II) Modify the `mirror` program to correct the rays that are drawn incorrectly when the maximum ray height is close to the mirror radius (see exercise 2). Run your program and print out the resulting graphs for the following values of maximum ray height: (a) 8.0; (b) 9.0 and (c) 10.0.
4. (II) Modify the `mirror` program to draw the rays coming from a point source located at the center of curvature of the mirror. Run your program and print out the resulting graph for a mirror which consists of a 90° circular arc (spanning from -45° to 45°).
5. (II) Modify the `mirror` program to draw the rays for a convex mirror (see Figure 37–7 in your textbook). Run your program and print out the resulting graphs for the cases listed in exercise 1.

6. (III) Modify the `mirror` program to make the rays emanate from a point source located at an arbitrary location. Have the spherical mirror consist of an arc spanning from -45° to 45° with the center of curvature at the origin. Run your program and print out the resulting graphs for the following locations of the point source: (a) (0,0); (b) (2,0); (c) (5,0); (d) (0,2) and (e) (0,5).

7. (III) Modify the `mirror` program to draw the rays for a parabolic mirror with a focal point located at $x = 5.0$. Run your program and print out the resulting graphs for the cases listed in exercise 1.

Listing

`mirror.m`

```
% mirror - Program to compute simple reflection from a
%           spherical convex mirror
clear all; help mirror; % Clear memory and print header

%@ Initialize variables (e.g., mirror radius, number of rays)
Radius = 10.0; % Radius of the mirror
fprintf('Radius of the mirror is %g \n',Radius);
x0 = -Radius; % X-coordinate of source rays
y0Max = input('Enter the max initial height of rays: ');
NRays = input('Enter the number of rays to draw: ');

%@ Set up for plotting rays
clf; figure(gcf); % Clear figure window and bring it forward
plot(0.0,0.0,'ro'); % Mark the origin
axis([-Radius, Radius, -Radius, Radius]);
axis('equal'); % Use square aspect ratio
xlabel('x'); ylabel('y');
title(sprintf('Spherical mirror; radius = %g',Radius));
hold on;

%@ Draw the mirror surface
NMirror = 100;
for i=1:NMirror
    yMirror(i) = -y0Max + (i-1)/(NMirror-1)*(2*y0Max);
    xMirror(i) = sqrt(Radius^2 - yMirror(i)^2);
end
plot(xMirror,yMirror,'b--','Erasemode','none');
drawnow;

%@ Loop over the rays to be drawn
for iRay=1:NRays

    %@ Determine starting point of the ray
    y0 = -y0Max + (iRay-1)/(NRays-1)*(2*y0Max);
    xRay(1) = x0; yRay(1) = y0; % First point

    %@ Find the ray's point of intersection with the mirror
    y1 = y0; % Ray enters horizontally
    x1 = sqrt(Radius^2 - y1^2);
    xRay(2) = x1; yRay(2) = y1; % Second point

    %@ Compute incident and reflection angles (relative to x-axis)
    ThetaI = atan2(y1,x1);
    ThetaR = 2*ThetaI;

    %@ Compute ray's final point back on vertical plane to the left
    x2 = x0;
```

```
y2 = (x2-x1)*tan(ThetaR) + y0;
xRay(3) = x2; yRay(3) = y2;      % Third point

%@ Draw the ray using its three points
plot(xRay,yRay,'-', 'EraseMode', 'none');
drawnow;
pause(1); % Pause 1 second between rays

end
hold off;
```